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The Unique Solvability of a Certain Nonlocal Nonlinear Problem with a Spatial Operator Strongly Monotone with Respect to the Gradient

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Abstract—We consider a nonlinear degenerate parabolic equation whose spatial operator depends on a nonlocal characteristic of the solution. We prove the uniqueness of the solution in the class of vector-valued functions that take on values in Sobolev spaces.

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The result obtained in this paper is a generalization of the uniqueness theorem proved in [1] for a parabolic equation whose spatial operator is nonlocal, but linear with respect to the gradient.

The existence of a solution to the problem under consideration follows from results of [2], where one proves the existence of a solution to a doubly degenerate equation with a nonlocal operator in a more general form.

1. THE PROBLEM

LetΩ be a bounded domain in R^n with the boundary Γ, $Q_T = \Omega \times (0,T)$. Consider the initial-boundary value problem

$$\frac{\partial u}{\partial t} - \sum_{i=1}^{n} \frac{\partial}{\partial x_i} k_i(x, \nabla u, Bu) = f, \quad (x,t) \in Q_T, \tag{1}$$

$$u(x,0) = u_0(x), \quad x \in \Omega, \quad u|_\Gamma = 0. \tag{2}$$

Here k_i are given functions, ∇u is the gradient of u, and B is an operator in the form

$$Bu(t) = \int_{\Omega'} g(x, u(x,t))\, dx,$$

where $g(x,\xi_0)$, $x \in \Omega'$, $\xi_0 \in R$ is a known function, and Ω' is a domain belonging to Ω or coinciding with it.

Equations in form (1) emerge, for example, in the mathematical description of the diffusion of bacterial population under the assumption that the speed of spreading at a point is defined by the global state of the environment [1, 3].

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