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A Majorant-Minorant Criterion for the Total Preservation of Global Solvability of a Functional Operator Equation

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Abstract—We study a nonlinear controlled functional operator equation in an ideal Banach space. We establish sufficient conditions for the global solvability for all controls from a given set, and obtain a pointwise estimate for solutions. Using upper and lower estimates of the functional component in the right-hand side of the initial equation (with a fixed operator component), we obtain majorant and minorant equations. We prove the stated theorem, assuming the monotonicity of the operator component in the right-hand side and the global solvability of both majorant and minorant equations. We give examples of the reduction of controlled initial boundary value problems to the equation under consideration.

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INTRODUCTION

Let us use the following denotations: $n, m, \ell, s \in \mathbb{N}$ are given numbers, $\Pi \subset \mathbb{R}^n$ is a measurable (hereinafter in the Lebesgue sense) bounded set, $\mathcal{X}$, $\mathcal{Z}$, and $\mathcal{U}$ are *Banach ideal spaces*[1)] (BIS) of measurable on Π functions, $\mathcal{D} \subset \mathcal{U}^s$ is a given set, and $A : \mathcal{Z}^m \to \mathcal{X}^\ell$ is a given *linear bounded operator* (LBO). Further for the vector function $x \in \mathcal{X}^\ell$, representing the image of a vector function $z \in \mathcal{Z}^m$ under the mapping realized by the operator A, we use an equivalent denotation; namely, depending on a situation, we write $x = A[z]$; $x(t) = A[z](t)$, $t \in \Pi$; $x(\cdot) = A[z](\cdot)$; or $x = A\,[z(\cdot)]$. Consider the controlled functional operator equation

$$x(t) = \theta(t) + A\big[f(\cdot, x(\cdot), u(\cdot))\big](t), \quad t \in \Pi, \quad x \in \mathcal{X}^\ell, \tag{1}$$

where $u \in \mathcal{D}$ is a control, $\theta \in \mathcal{X}^\ell$, $f(t, x, v) : \Pi \times \mathbb{R}^\ell \times \mathbb{R}^s \to \mathbb{R}^m$ is a given function measurable with respect to $t \in \Pi$, continuous with respect to $\{x, v\} \in \mathbb{R}^\ell \times \mathbb{R}^s$, and such that

$\mathbf{F}_1$) for all $y \in \mathcal{X}^\ell$ and $u \in \mathcal{D}$ the superposition $f\big(\cdot, y(\cdot), u(\cdot)\big)$ belongs to $\mathcal{Z}^m$;

$\mathbf{F}_2$) for any $u \in \mathcal{D}$ and $y_* \in \mathcal{X}_+$ there exists a constant $\mathcal{N}[y_*, u] > 0$ such that

$$\big\|f(\cdot, y(\cdot), u(\cdot)) - f(\cdot, z(\cdot), u(\cdot))\big\|_{\mathcal{Z}^m} \le \mathcal{N}[y_*, u]\, \|y - z\|_{\mathcal{X}^\ell} \;\; \forall y, z \in \mathcal{X}^\ell, \;\; |y|, |z| \le y_*.$$

In [1–3] one has shown (see also examples in Section 3) that the *method of inversion of the main part* of a differential equation enables one to reduce a rather wide class of controlled *initial boundary value problems* (IBVP) to Eq. (1). Due to this fact, one can use this equation as a tool for studying various issues in the theory of controlled distributed systems, in particular, the global solvability problem.

The violation of the global solvability of a distributed system is rather probable when the growth order of the right-hand side of the corresponding differential equation with respect to the phase variable exceeds the linear one. In [4] (see the example for theorem 2.2, pp. 87–88; § 4, pp. 95–100), [5] (§ 1),

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1)Recall that a Banach space E of measurable functions is said to be a Banach ideal space if $\{y \in E,\ x$ is a measurable function, $|x| \le |y|\} \Longrightarrow \{x \in E, \|x\|_E \le \|y\|_E\}$.