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Types of Maximum Noninteger Vertices of the Relaxation Polyhedron of the Four-Index Axial Assignment Problem

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Abstract—We describe various types of maximum noninteger vertices. We identify types of polyhedron vertices by the number of fractional components contained in three-sections of four-index matrices representing the polyhedron vertices.

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It is known [1] that the p-index ($p \geq 3$) axial assignment problem (which has many applications [2–4]) is NP-complex. In order to elaborate efficient algorithms for its solution, one studies the relaxation polyhedron

$$M(p,n) = \Bigg\{ x = \|x_{i_1 i_2 \ldots i_p}\|_n : x_{i_1 i_2 \ldots i_p} \geq 0 \, \forall (i_1, i_2, \ldots, i_p) \in N_n^p, \\ \sum_{i_1=1}^{n} \ldots \sum_{i_{s-1}=1}^{n} \sum_{i_{s+1}=1}^{n} \ldots \sum_{i_p=1}^{n} x_{i_1 i_2 \ldots i_p} = 1 \ \ \forall i_s \in N_n, \ \ \forall s \in N_p \Bigg\}$$

(where $N_n = \{1, 2, \ldots, n\}$, N_n^p is the Cartesian product of p sets, each of which equals N_n) generated by conditions of the problem ([5], P. 308).

In [6–9] one has proposed an approach to the characterization of noninteger vertices of the relaxation polyhedron $M(3,n)$ of the three-index axial assignment problem based on introduced notions of an "r-noninteger vertex" and a "completely r-noninteger vertex".

With the help of this approach in [10–13] one has proved important theorems allowing one to characterize all types of completely r-noninteger vertices and to advance essentially in studying the structure of noninteger vertices and in establishing combinatorial properties of the polyhedron $M(3,n)$. One identifies types of vertices of the polyhedron $M(3,n)$, taking into account the number of fractional components that are contained in two-dimensional sections of three-index matrices which represent its vertices.

The problem of classification of all types of completely r-noninteger vertices (even maximum noninteger vertices) of the polyhedron $M(p,n)$ with $p \geq 4$ is not completely solved yet. In this connection one tries to develop methodological bases for its solution.

Continuing the research in the indicated area (see [10–13] for the case $p = 3$), we consider questions connected with the description of new types of maximum noninteger vertices (those with $4n-3$ fractional components) of the relaxation polyhedron $M(4,n)$, $n \geq 3$.

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