ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2011, Vol. 55, No. 10, pp. 78–81. © Allerton Press, Inc., 2011.
Original Russian Text © M.A. Aukhadiev, S.A. Grigoryan, and E.V. Lipacheva, 2011, published in Izvestiya Vysshikh Uchebnykh Zavedenii. Matematika, 2011, No. 10, pp. 89–93.

A Compact Quantum Semigroup Generated by an Isometry

M. A. Aukhadiev*, S. A. Grigoryan, and E. V. Lipacheva*****
(Submitted by D.Kh. Mushtari)

Kazan State Energy University, ul. Krasnosel'skaya 51, Kazan, 420066 Russia
Received March 23, 2011

Abstract—In this paper we construct a compact quantum semigroup structure on a Toeplitz algebra. We prove the existence of a subalgebra in the dual algebra isomorphic to the algebra of regular Borel measures on a circle with the convolution product. We also prove the existence of Haar functionals in the dual algebra and in the mentioned subalgebra. We show that this compact quantum semigroup contains a dense subalgebra with the structure of a weak Hopf algebra.

DOI: 10.3103/S1066369X11100100

Keywords and phrases: *quantum semigroup, quantum group, Haar functional, Toeplitz algebra, weak Hopf algebra, inverse semigroup.*

1. THE NECESSARY INFORMATION

Let

$$l^2(\mathbb{Z}_+) = \left\{ f : \mathbb{Z}_+ \to \mathbb{C},\ \sum_{n=0}^{\infty} |f(n)|^2 < \infty \right\}$$

be the Hilbert space of all functions square summable on the set of nonnegative integers. The family of functions $\{e_n\}_{n=0}^{\infty}$, $e_n(m) = \delta_{n,m}$, where $\delta_{n,m}$ is the Kronecker symbol, forms an orthonormal basis in $l^2(\mathbb{Z}_+)$. A bounded linear operator $T : l^2(\mathbb{Z}_+) \to l^2(\mathbb{Z}_+)$ is said to be the right shift operator, if

$$Te_n = e_{n+1},\ \ n \in \mathbb{Z}_+.$$

Evidently, T is an isometric operator. The left shift operator T^*:

$$T^*e_0 = 0,\ \ T^*e_n = e_{n-1},\ \ n > 0,$$

is conjugate to it. A Toeplitz algebra is a uniformly closed subalgebra of the algebra $\mathcal{B}(l^2(\mathbb{Z}_+))$ of all bounded linear operators on $l^2(\mathbb{Z}_+)$ generated by operators T and T^*. There exist various generalizations of a Toeplitz algebra (e.g., [1–3]). In what follows we denote a Toeplitz algebra by $\mathcal{T}$.

Since $T^*T = 1$ and TT^* is a projector, each finite product of operators T and T^* coincides with the operator

$$T^n T^{*m} = T_{n,m},\ \ n, m \in \mathbb{Z}_+.$$

Therefore in a Toeplitz algebra $\mathcal{T}$ finite linear combinations $\sum\limits_{n,m\in\mathbb{Z}_+} \lambda_{n,m} T_{n,m}$, $\lambda_{n,m} \in \mathbb{C}$, are dense with respect to the uniform topology. Evidently,

$$T_{n,m}T_{k,l} = \begin{cases} T_{n+k-m,l}, & \text{if } k > m; \\ T_{n,l+m-k}, & \text{if } k < m; \\ T_{n,l}, & \text{if } k = m. \end{cases}$$

*E-mail: m.aukhadiev@gmail.com.
**E-mail: gsuren@inbox.ru.
***E-mail: elipacheva@gmail.com.