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The Existence of Fixed Points of Left-Continuous Monotone Operators in Spaces with a Regular Cone

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Abstract—In theorems on the existence of a fixed point of an operator the latter is usually assumed to be continuous. In this paper we prove a theorem with sufficient conditions for the existence of a fixed point of an operator which is not necessarily continuous (possibly it is left-continuous). The obtained theorem with the use of regular cones is applied for proving the existence of a fixed point of a nonlinear integral operator. We give an example illustrating the theorem.

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INTRODUCTION

In theorems on the existence of a fixed point of an operator the latter is usually assumed to be continuous. However there are problems reducible to operator equations with discontinuous operators. Some of these problems are described, for example, in the paper [1]. When considering an operator equation in a space with a partial order, one sometimes can refuse the continuity of the operator. For example, the Birkhoff—Tarski existence theorem ([2], P. 154, theorem 11) gives conditions for a partial ordering of a set and the monotonicity of an operator.

For operators acting in a Banach space semiordered with the help of a cone there are fixed point theorems which do not require the continuity of the operator (e.g., [3], pp. 128—129, theorem 4.1). In [4] one considers operator equations in a Banach space semiordered with the help of a cone. Along with the monotonicity, one also imposes the so-called lower continuity condition on the operator. It is shown that under certain conditions the solution to the equation is obtained as the limit of successive approximations.

Monotone iterative processes are used for the proof of the solvability of equations with continuous operators and for determining their solutions, for example, in papers [3] (Chap. 4, § 1) and [5] (§§ 3, 10).

In this paper we obtain new sufficient conditions for the existence of a fixed point of a left-continuous operator. The notion of the left continuity coincides with the continuity from below used in [4]. We apply the obtained existence theorem for proving the existence of a fixed point of a nonlinear integral operator.

1. CONES AND MONOTONE OPERATORS

Let us recall some notions mentioned in the paper [3].

Let X be a real Banach space. A cone in the Banach space X is a set K such that

1) K is closed;

2) $u, v \in K, \alpha, \beta \geqslant 0 \Rightarrow \alpha u + \beta v \in K$;

3) $x \in K, x \neq \Theta \Rightarrow -x \notin K$, where Θ is the zero of the space X.

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