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Solution Approximation in a Minimax Control Problem for a Singularly Perturbed System with Delay

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Abstract—We consider a control problem with a minimax criterion for a singularly perturbed system with delay. We propose a procedure for constructing an initial approximation to control in the minimax control problem.

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INTRODUCTION

In this paper we consider dynamic objects whose mathematic models are singularly perturbed systems (with a small parameter at some derivatives) with a delay (with respect to a state). The solution of optimization problems for such objects is based on various asymptotic representations of their trajectories. The most used approach implies the decomposition of the boundary-value problem (obtained from the principle of maximum) on the base of the method of boundary functions. Many recent papers are devoted to optimal control problems for such systems (reviews [1–3]). The dependence of the current change rate of output variables of the system on their values at previous time moments is described by differential equations with aftereffect [4].

Consider the optimal control problem stated in [5, 6] for singularly perturbed systems with delay under the uncertainty in the initial conditions. The terminal quality functional depends both on fast and slow variables. The proposed method is based on the idea of the representation of the fundamental matrix of solutions (partitioned into blocks in accordance with dimensions of fast and slow variables) in the form of a uniformly converging sequence [7]. The realization of this method is based on results obtained in [5–11] and on the apparatus of convex analysis [12]. We construct an initial approximation to the optimal solution (with respect to a small parameter) without additional requirements of the smoothness of feasible controls (we assume only that they are at least once differentiable).

1. THE PROBLEM

Consider the following controllable singularly perturbed system with a small parameter $\mu > 0$ (at some derivatives) with a delay $h > 0$ (with respect to the state):

$$\begin{aligned} dx(t)/dt &= A_{11}(t)x(t) + A_{12}(t)y(t) + G_1(t)x(t-h) + B_1(t)u(t), \\ \mu dy(t)/dt &= A_{21}(t)x(t) + A_{22}(t)y(t) + G_2(t)x(t-h) + B_2(t)u(t), \end{aligned} \tag{1.1}$$

where $t \in T = [t_0, t_1]$; $x \in R^n$, $y \in R^m$; A_{ij}, B_i, and G_i, $i, j = 1, 2$, are matrices of the corresponding dimensions with continuous elements. The initial state of the system $x(t) = \psi(t)$, $t_0 - h \le t < t_0$, $x(t_0) = x_0$, $y(t_0) = y_0$ is not exactly known, given are only constraints $x_0 \in X_0$ and $y_0 \in Y_0$, where X_0 and Y_0 are convex compacts in the corresponding spaces, $\psi(t) \in \Psi(t)$, $t_0 - h \le t < t_0$, $\Psi(t)$ is a given multivalued mapping whose values are convex compacts (in R^n); $\Psi(t)$ is assumed to be continuous with respect to t in the Hausdorff metric. Realizations of the control $u(t)$, $t \in T$, are Lebesgue

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