Connection of Some Bilevel and Nonlinear Optimization Problems

A. M. Malyshev* and A. S. Strekalovskii**

(Submitted by Ya.I. Zabotin)

Institute of Dynamic Systems and Control Theory, of Siberian Branch of Russian Academy of Sciences,
ul. Lermontova 134, Irkutsk, 664033 Russia

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Abstract—In this paper we reduce a quadratic-linear bilevel optimization problem with a guaranteed solution to a family of bilevel problems in the optimistic statement. Then we reduce the obtained bilevel problems to nonconvex one-level optimization problems and solve the latter by nonconvex optimization methods.

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1. INTRODUCTION

Various applied problems [1–4] which arise in modeling hierarchical control systems in energetics, economics, ecology, etc. [2] lead to bilevel optimization problems. Usual solutions to the latter problems are optimistic (cooperative) and guaranteed (non-cooperative, pessimistic) ones [1–4]. In the first case one assumes that the interest of the upper level can be coordinated with the actions of the lower level. In the second case such a coordination of interests is absent.

In the last three decades many papers have been devoted to the search of optimistic solutions to bilevel problems [5], and only few ones have been dedicated to the search of a guaranteed solution [6–8]. Note that in bilevel problems even the search of an optimistic solution (which is simpler) is reduced, as usual, to the search of global solutions to nonconvex optimization problems [4], and therefore from the computational point of view it represents a complex problem [9–12]. Here we reduce the considered bilevel problem with a guaranteed solution to a nonconvex optimization problem.

2. PROBLEMS AND THEIR INTERCONNECTIONS

Consider the following quadratic-linear bilevel optimization problem:

\[ W(x, \varepsilon) \triangleq \sup_{y} \{ F(x, y) \mid y \in Y_{*}(x, \varepsilon) \} \downarrow \min_{x}, \tag{BP(\varepsilon)} \]

where \( X \triangleq \{ x \in \mathbb{R}^{m} \mid Ax \leq a, x \geq 0 \} \), \( Y(x) \triangleq \{ y \in \mathbb{R}^{n} \mid A_{1}x + B_{1}y \leq b, y \geq 0 \} \), \( F(x, y) \triangleq \frac{1}{2}(x, Cx) + \langle c, x \rangle - \frac{1}{2}(y, C_{1}y) + \langle c_{1}, y \rangle \), \( G(y) \triangleq \langle d, y \rangle \), \( A \in \mathbb{R}^{p \times m} \), \( A_{1} \in \mathbb{R}^{q \times m} \), \( B_{1} \in \mathbb{R}^{q \times n} \), \( C = C^{T} \geq 0 \), \( C_{1} = C_{1}^{T} \geq 0 \), \( c \in \mathbb{R}^{m} \), \( c_{1}, d \in \mathbb{R}^{q} \), \( a \in \mathbb{R}^{p} \), \( b \in \mathbb{R}^{q} \), and \( Y_{*}(x, \varepsilon) \) is the set of \( \varepsilon \)-solutions to the lower level problem.

Here \( F(x, y) \) and \( G(y) \) are efficiency criteria of players of the upper and lower levels, therefore \( W(x, \varepsilon) = \sup_{y} \{ F(x, y) \mid y \in Y_{*}(x, \varepsilon) \} \) is an estimate of the efficiency of the strategy of the upper level player [1, 2].

*E-mail: anton@irk.ru.
**E-mail: strekal@icc.ru.