ISSN 1066-369X, Russian Mathematics (Iz. VUZ), 2007, Vol. 51, No. 1, pp. 71–77. © Allerton Press, Inc., 2007.
Original Russian Text © V. F. Chistyakov, E. V. Chistyakova, 2007, published in Izvestiya Vysshykh Uchebnykh Zavedenii. Matematika, 2007, No. 1, pp. 76–81.

Nonlocal Theorems on Existence of Solutions of Differential-Algebraic Equations of Index 1

V. F. Chistyakov and E. V. Chistyakova

Institute of System Dynamics and Control Theory, Siberian Branch of the Russian Academy of Sciences, POB 1233, ul. Lermontova 134, Irkutsk, 664033 Russia
E-mail: chistyak@gmail.com

Received January 11, 2005

DOI: 10.3103/S1066369X07010094

We consider the initial value problem

$$A(t)\dot{x} + B(x,t) = 0, \;\; t \in T = [t_0, t_1], \tag{0.1}$$

$$x(t_0) = x_0, \tag{0.2}$$

where $A(t)$ is an $n \times n$-matrix, $B(x,t)$ an n-dimensional vector-function, $x \equiv x(t)$ the unknown vector-function, $(x(t),t) \in U = R^n \times T$, x_0 a given vector from R^n, $\dot{} \equiv d/dt$. We assume that

$$\det A(t) = 0 \;\; \forall t \in T \tag{0.3}$$

and that the input data are of the following smoothness: $A(t) \in C^1(T)$, $B(x,t) \in C^2(U)$.

Systems of the form (1) satisfying condition (3) are called differential-algebraic equations (DAE) [1]. Such systems appear in many areas of applications, e. g., in the analysis of electronic circuits of electrical networks and mechanical constructions [1]–[4]. At present, a series of existence theorems for solutions of problems of the form (1), (2) have been proved, but, in the case of a nonlinear vector-function $B(x,t)$, all these theorems are local ([2], p. 160; [3], p. 36), whereas there is a wide class of problems which require nonlocal statements. In particular, for systems $A(t)\dot{x} + B(x,t,u) = 0$, where $u \equiv u(t)$ is a control, on the statement of problems on controllability and observability, one needs the assumption of extendability of a solution to the whole segment T. In addition, nonlocal theorems are important for numerical solution of DAE's since they give the certainty concerning the absence of singular points on the segment of definition of the system. For example, in [4], an initial value problem is considered which has a known solution $\forall t \in (-\infty, +\infty)$, however, at points $\sqrt{\pi k}$, $k = 1, 2, \dots$, from this solution, unknown solutions branch off, and on numerical calculations an emergency stop takes place if the segment T contains points $\sqrt{\pi k}$.

For systems in normal form (in (1), $A(t) = E_n$), nonlocal theorems are based either on the assumption that the vector-function $B(x,t)$ is globally Lipschitzian ([5], p. 392) or on some divisions of the stability theory including the La Salle theorem ([6], p. 276). Here and in what follows E_* is the identity matrix of dimension equal to the index. There are also more complicated methods based on the study of the spectrum of the operator of the Jacobi matrix of the vector-function $B(x,t)$ ([5], Pevzner's theorem). In this paper, we use analogues of the first two approaches.

Definition 5. A differential operator $\Lambda_l = \sum_{j=0}^{l} W_j(t,Z)(d/dt)^j$, where $Z = (x, \dot{x}, \dots, x^{(l+1)})$, $W_j(t,Z)$ are smooth $n \times n$-matrices, is called a left regularizing operator (LRO) for system (1) if

$$\Lambda_l \circ [A(t)\dot{x} + B(x,t)] = \mathcal{A}(x,t)\dot{x} + \mathcal{B},(x,t) \;\; \forall x \in C^{l+1}(T), \;\; (x(t),t) \in U,$$

and $\det \mathcal{A}(x_0,t_0) \neq 0$. The minimal possible number l is called the index of the system.

Definition 6 ([1], p. 31)**.** A semi-inverse matrix of an $m \times n$-matrix A is an $n \times m$-matrix A^- satisfying the matrix equation $AA^-A = A$.