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Solution of Boundary Value Problems for One Degenerate B-Elliptic Equation of the 2nd Kind by the Method of Potentials

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Let E_2^+ stand for the first quadrant of the Cartesian plane Oxy; let D^+ be a finite domain in E_2^+ bounded by a curve Γ^+ and segments Γ_1 and Γ_0 of the coordinate axes Ox and Oy; $D_e^+ = E_2^+/\overline{D}^+$.

In [1] we study boundary value problems for a degenerate B-elliptic equation in the form

$$L(u) = y^m B_x u + \frac{\partial^2 u}{\partial y^2} = 0, \tag{L}$$

where $B_x = \frac{\partial^2}{\partial x^2} + \frac{k}{x}\frac{\partial}{\partial x} = x^{-k}\frac{\partial}{\partial x}\left(x^k \frac{\partial}{\partial x}\right)$ is the Bessel operator, $m > 0$, $k > 0$ are constant values. In this paper, we study the main boundary value problems for the degenerate B-elliptic equation

$$E_B(u) = B_x u + y^m \frac{\partial^2 u}{\partial y^2} = 0, \tag{E_B}$$

where $m > 4$, $k > \frac{m}{m-2}$ are constant values.

For equations (L) and (E_B), $y = 0$ is the line of the parabolic degeneration. With $y > 0$ equations (L) and (E_B) are elliptic. In [2] one defines B-elliptic equations as elliptic ones such that the Bessel operator acts with respect to one of the variables. Degenerate B-elliptic equations (L) and (E_B) are notable in the fact that the line of the parabolic degeneration $y = 0$ is not a characteristic line for equation (L), but it is a characteristic line for equation (E_B). Equations (L) and (E_B) are called degenerate B-elliptic equations of the first and second kind, respectively. One can use the results of this paper in the theory of boundary value problems for degenerate elliptic equations with many variables and axially symmetric problems in the theory of potential, arising from important applied problems [3]–[7].

1. THE FUNDAMENTAL SOLUTION

In characteristic coordinates

$$\xi = x, \quad \eta = \frac{2}{2-m} y^{\frac{2-m}{2}}$$

equation (E_B) takes the form

$$B_\xi u + \frac{\partial^2 u}{\partial \eta^2} + \frac{m}{m-2}\eta^{-1}\frac{\partial u}{\partial \eta} = 0. \tag{1.1}$$

We seek for a solution to equation (1.1) in the form

$$u = \eta^{\frac{2}{2-m}} v, \tag{1.2}$$

where v is a new unknown. Inserting this function in (1.1), we obtain the following equation with respect to v:

$$B_\xi v + \frac{\partial^2 v}{\partial \eta^2} + \frac{m-4}{m-2}\eta^{-1}\frac{\partial v}{\partial \eta} = 0. \tag{1.3}$$