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On Game Problems for Second-Order Evolution Equations

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1. In this paper, we consider certain problems of the theory of differential games in systems with distributed parameters. The players influence on the system with the use of control parameters contained in the right-hand side of the equation. Controls of players are chosen in the form of functions on which various constraints are imposed, so-called geometric, integral, and mixed constraints. Note papers [1]–[14] devoted to this research area.

In the first three games, the goal of the first player is to bring the system into an unperturbed state. In the fourth game, the goal of the first player is to bring the system and its velocity into an arbitrary ℓ-neighborhood of zero. The second player in all the games has the opposite goal. We present conditions (see below) which are sufficient in order that the first player can reach the goal in a finite time. For the third game, we also consider the encounter-evasion problem (see Proposition in Item 4).

Consider in the space $L_2(\Omega)$, where Ω is a bounded domain with piece-wise smooth boundary in R^n, a differential operator A of the form

$$Az = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i}\left(a_{ij}(x)\frac{\partial z}{\partial x_j}\right), \;\; x \in \Omega, \;\; a_{ij} \in C^1(\overline{\Omega}). \tag{0.1}$$

The domain of definition $D(A)$ of A is the space $\overset{\circ}{C}{}^2(\Omega)$ of twice continuously differentiable finite functions. The functions $a_{ij}(x)$, $x \in \Omega$, satisfy the following conditions: $a_{ij}(x) = a_{ji}(x)$, $x \in \Omega$, there exists a constant $\gamma \neq 0$ such that

$$\sum_{i,j=1}^{n} a_{ij}(x)\xi_i\xi_j \geq \gamma^2 \sum_{i=1}^{n} \xi_i^2 \tag{0.2}$$

for any $\xi = (\xi_1, \dots, \xi_n) \in R^n$ and $x \in \Omega$.

Let $(z, y)_A = (Az, y)$ for $z, y \in \overset{\circ}{C}{}^2(\Omega)$.

One can easily check that $(\cdot,\cdot)_A$ is a scalar product and $\overset{\circ}{C}{}^2(\Omega)$ is an incomplete Hilbert space. Completing it with respect to the norm $\|z\|_A = \sqrt{(Az, z)}$, $z \in \overset{\circ}{C}{}^2(\Omega)$, we obtain a complete Hilbert space associated with the operator A.

It is known that the operator A satisfying condition (2) has a discrete spectrum. In more exact terms, it has an infinite sequence $\lambda_1, \lambda_2, \dots$ of positive nondecreasing eigenvalues with limit at infinity and a sequence of generalized eigenfunctions $\varphi_1, \varphi_2, \dots$ which form a complete orthonormal system in the space $L_2(\Omega)$ ([15], p. 98).

Denote by $C(0, T; H_r(\Omega))$ ($L_2(0, T; H_r(\Omega))$) the space consisting of continuous (measurable) functions defined on $[0, T]$ and taking values in $H_r(\Omega)$, where r is a nonnegative number, T a positive number, and

$$H_r(\Omega) = \left\{ f \in L_2(\Omega) : f = \sum_{i=1}^{\infty} \alpha_i\varphi_i, \quad \sum_{i=1}^{\infty} \lambda_i^r \alpha_i^2 < \infty \right\}.$$

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