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The Jump Problem and the Faber–Schauder Series

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1. INTRODUCTION

This paper is dedicated to the solvability conditions for the jump problem on a nonrectifiable open arc. For simplicity, we define this arc as follows. Let a continuous real function $Y(x)$ be defined on the segment $I = [0, 1]$ and equal zero at its end points. Its graph

$$\Gamma = \{z = x + iY(x) : 0 \leq x \leq 1\} \tag{1.1}$$

is a simple arc on the complex plane C, originating at the point 0 and ending at 1. The jump problem implies the determination of a holomorphic in $\overline{C} \setminus \Gamma$ function $\Phi(z)$, according to the edge condition

$$\Phi^+(t) - \Phi^-(t) = f(t), \quad t \in \Gamma'; \tag{1.2}$$

here $\Phi^{\pm}(t)$ are the limit values of $\Phi(z)$ at the point $t \in \Gamma$ for z tending to t from above and from below, correspondingly, and $\Gamma' = \Gamma \setminus \{0, 1\}$. We assume that the function f defined on Γ satisfies the Hölder condition

$$\sup\left\{\frac{|f(t') - f(t'')|}{|t' - t''|^\nu} : t', t'' \in \Gamma,\ t' \neq t''\right\} \equiv h_\nu(f, \Gamma) < \infty$$

with a certain exponent $\nu \in (0, 1]$. In what follows, $H_\nu(\Gamma)$ stands for the set of all functions defined on Γ which satisfy this condition.

The jump problem represents an important partial case of the Riemann boundary value problem. In this connection, many well-known results on its solvability are obtained (see [1], [2]). However, these results, mainly, concern the case of a piecewise-smooth arc Γ, when a solution is defined by the Cauchy integral along this arc.

Later in [3] one obtained some results on the solvability of the jump problem on nonrectifiable curves. Namely, the following fact was proved. For any simple arc Γ and any function $f \in H_\nu(\Gamma)$ defined on it, under the condition

$$\nu > \frac{1}{2} \operatorname{dm} \Gamma \tag{1.3}$$

a holomorphic in $\overline{C} \setminus \Gamma$ function $\Phi(z)$ exists, whose boundary values are connected by equality (2). Here $\operatorname{dm} \Gamma$ is the well-known in the fractal theory upper metric dimension of Γ (e. g., [4], p. 37).

For $Y \in H_\mu(I)$ the condition

$$\nu > 1 - \frac{\mu}{2} \tag{1.4}$$

also guarantees the solvability of the jump problem (see [3]). As was shown in [3], condition (3) is unimprovable in terms of the values ν and $\operatorname{dm} \Gamma$. Condition (4), in turn, is unimprovable in terms of the exponents μ and ν; one can prove this fact, slightly modifying the example adduced in [5]. However, one can extend the classes of the curves, for which the jump problem is solvable, using other their