The Jump Problem and the Faber–Schauder Series

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1. INTRODUCTION

This paper is dedicated to the solvability conditions for the jump problem on a nonrectifiable open arc. For simplicity, we define this arc as follows. Let a continuous real function $Y(x)$ be defined on the segment $I = [0, 1]$ and equal zero at its end points. Its graph

$$
\Gamma = \{ z = x + iy(x) : 0 \leq x \leq 1 \}
$$

(1.1)
is a simple arc on the complex plane $\mathbb{C}$, originating at the point 0 and ending at 1. The jump problem implies the determination of a holomorphic in $\mathbb{C} \setminus \Gamma$ function $\Phi(z)$, according to the edge condition

$$
\Phi^+(t) - \Phi^-(t) = f(t), \quad t \in \Gamma';
$$

(1.2)

here $\Phi^\pm(t)$ are the limit values of $\Phi(z)$ at the point $t \in \Gamma$ for $z$ tending to $t$ from above and from below, correspondingly, and $\Gamma' = \Gamma \setminus \{0, 1\}$. We assume that the function $f$ defined on $\Gamma$ satisfies the Hölder condition

$$
\sup \left\{ \frac{|f'(t') - f'(t'')|}{|t' - t''|^{\nu}} : t', t'' \in \Gamma, t' \neq t'' \right\} \equiv h_\nu(f, \Gamma) < \infty
$$

with a certain exponent $\nu \in (0, 1]$. In what follows, $H_\nu(\Gamma)$ stands for the set of all functions defined on $\Gamma$ which satisfy this condition.

The jump problem represents an important partial case of the Riemann boundary value problem. In this connection, many well-known results on its solvability are obtained (see [1], [2]). However, these results, mainly, concern the case of a piecewise-smooth arc $\Gamma$, when a solution is defined by the Cauchy integral along this arc.

Later in [3] one obtained some results on the solvability of the jump problem on nonrectifiable curves. Namely, the following fact was proved. For any simple arc $\Gamma$ and any function $f \in H_\nu(\Gamma)$ defined on it, under the condition

$$
\nu > \frac{1}{2} \text{dim} \Gamma
$$

(1.3)
a holomorphic in $\mathbb{C} \setminus \Gamma$ function $\Phi(z)$ exists, whose boundary values are connected by equality (2). Here $\text{dim} \Gamma$ is the well-known in the fractal theory upper metric dimension of $\Gamma$ (e.g., [4], p. 37).

For $Y \in H_\mu(I)$ the condition

$$
\nu > 1 - \frac{\mu}{2}
$$

(1.4)
also guarantees the solvability of the jump problem (see [3]). As was shown in [3], condition (3) is unimprovable in terms of the values $\nu$ and $\text{dim} \Gamma$. Condition (4), in turn, is unimprovable in terms of the exponents $\mu$ and $\nu$; one can prove this fact, slightly modifying the example adduced in [5]. However, one can extend the classes of the curves, for which the jump problem is solvable, using other their