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On Fundamental Equations of Almost Geodesic Mappings of Type $\pi_2(e)$

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1. INTRODUCTION

In [1]–[3] almost geodesic mappings between torsion-free affinely connected spaces A_n and $\overline{A}_n$ were introduced and three types of these mappings: π_1, π_2 and π_3 were specified. For $n > 5$, this classification was proved to be complete [4], [5]. The almost geodesic mappings were studied by many authors (see, e. g., [6]–[14]). In the present paper we study the fundamental equations of almost geodesic mappings of type $\pi_2(e)$, $e = \pm 1$.

Note that we do not impose any restrictions on the signature of metrics of Riemannian spaces V_n as done, for example, in [2] (p. 50) or [15] (p. 11). All our considerations are local, and all functions are assumed to be sufficiently smooth.

2. ALMOST GEODESIC MAPPINGS

A curve ℓ in an affinely connected space A_n is said to be *almost geodesic* if along ℓ there exists a two-dimensional parallel distribution containing the tangent vector of ℓ at each point.

A diffeomorphism $f : A_n \to \overline{A}_n$ is an *almost geodesic mapping* if any geodesic curve of A_n is mapped onto an almost geodesic curve of $\overline{A}_n$.

In [1]–[3], an almost geodesic mapping $f : A_n \to \overline{A}_n$ of type π_2 is called a *mapping of type* $\pi_2(e)$ if the inverse mapping $f^{-1} : \overline{A}_n \to A_n$ is also an almost geodesic mapping of type π_2.

A mapping $f : A_n \to \overline{A}_n$ is almost geodesic of type $\pi_2(e)$ if the deformation tensor of affine connections $P^h_{ij}(x) \equiv \overline{\Gamma}^h_{ij}(x) - \Gamma^h_{ij}(x)$ written with respect to the coordinate system $x \equiv (x^1, x^2, \dots, x^n)$ common with respect to f, satisfies ([1]; [2], p. 177; [3]):

$$\begin{aligned} &(a)\ P^h_{ij} = \delta^h_{(i}\psi_{j)} + F^h_{(i}\varphi_{j)},\\ &(b)\ F^h_{(i,j)} = F^h_{(i}\mu_{j)} + \delta^h_{(i}\varrho_{j)},\\ &(c)\ F^h_\alpha F^\alpha_i = e\delta^h_i, \quad e = \pm 1, 0, \end{aligned} \tag{1}$$

where Γ^h_{ij} ($\overline{\Gamma}^h_{ij}$) is the object of affine connection of the space A_n ($\overline{A}_n$), δ^h_i is the Kronecker delta, F^h_i is an affinor, ψ_i, φ_i, μ_i, ϱ_i are covectors, and (ij) stands for index symmetrization. Hereafter " , " denotes

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