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A MODIFICATION OF THE ROSEN METHOD FOR AN INVERSE-CONVEX PROBLEM

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1. Introduction

Let us consider the mathematical programming problem

$$h(x) \downarrow \min, \quad x \in S, \quad g(x) \geq 0, \tag{P}$$

where $h(\cdot)$, $g(\cdot)$ are convex functions on $\mathbb{R}^n$, S is a convex set in $\mathbb{R}^n$, $g(\cdot)$ is continuously differentiable on an open convex subset Ω of $\mathbb{R}^n$, $S \subset \Omega$, and $\{x \in \mathbb{R}^n \mid g(x) = 0\} \subset \Omega$. Let

$$\mathcal{V}(P) \triangleq h_* := \inf(h, D) \triangleq \inf\{h(x) \mid x \in S, \ g(x) \geq 0\} > -\infty. \tag{1}$$

The peculiarity of *Problem* (P) turns on the inverse-convex constraint $g(x) \geq 0$ which determines the complement of the open convex set $\{x \in \mathbb{R}^n \mid g(x) < 0\}$. It is natural to assume that $g(x_*) = 0$, where x_* is a solution of Problem (P), because without this assumption a solution of the relaxed problem

$$h(x) \downarrow \min, \quad x \in S, \tag{PW}$$

which is simpler than (P), is a solution of Problem (P).

With the help of the function $\mathcal{V}(P)$ defined in (1) one can propose another version of the requirement that the inverse-convex constraint is active at solutions of Problem (P):

$$\exists v \in S \ g(v) < 0 : h(v) < h_* \triangleq \mathcal{V}(P). \tag{2}$$

Condition (2) implies, in particular, that any optimization method [1] which solves Problem (PW) with an arbitrary initial point $x^0 \in S$ necessarily finds a point $\widetilde{x}$ such that $g(\widetilde{x}) = 0$, $h(v) < h(\widetilde{x}) < h(x^0)$. This procedure provides a descent onto the level surface $g(x) = 0$.

One of the local search methods is related to the global optimality conditions [2] and implies finding a point at the level surface $g(x) = 0$ and the sequential solution of the linearized problems:

$$(LQ(u, \beta)) : \ \langle \nabla g(u), x \rangle \uparrow \max, \quad x \in S, \quad h(x) \leq \beta, \tag{3}$$

which, obviously, are the convex programming problems. Here u and β are parameters of problem (3) [2].

In [2], this method is called a special local search method (SLSM). It enables us to find an approximate stationary point for the associated convex maximization problem (for certain β) [3]

$$(Q(\beta)) : \ g(x) \uparrow \max, \quad x \in S, \quad h(x) \leq \beta,$$

rather than for the initial Problem (P).

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