Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 49, No. 12, pp.59–64, 2005 *UDC 517.977*

A BILINEARIZATION SOLUTION METHOD FOR PROBLEMS OF PROGRAM CONTROL OPTIMIZATION

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1. Introduction. Problem definition

The construction, analysis, and realization of computational optimal control methods are studied in many papers (for example, [1]–[10]). The variational peculiarity of optimal control problems calls forth a variety of concepts, approaches, and numerical solution techniques. Let us define a wide class of methods which meet the following standard scheme:

1) the construction of an auxiliary problem for the definition of a descent direction (an auxiliary control);

2) the variation of the initial control on the base of the auxiliary one with the use of certain parameters;

3) the search of the variation parameters aimed at the decrease of the quality functional.

Within the optimal control problems, the implementation of this scheme at steps 1, 2 admits several versions (weak, needle, phase approximations of functionals, together with the corresponding variation). As a rule, the auxiliary problem for most methods does not contain variation parameters, i. e., steps 1, 2 are independent.

Nevertheless, the methods, where steps 1, 2 are united, i. e., whose auxiliary problem includes the variation procedure, together with the parameters, are of interest. This is connected with the fact that one solves the auxiliary problem at a certain admissible neighborhood of the nominal control which is constructed in a parametric form. The appropriateness of the local solution is quite clear, because any approximation, generally speaking, represents an adequate model of the initial functional only in a certain neighborhood of the process under consideration. In addition, the third step of the mentioned scheme implies the search of the admissible neighborhood by the variation of parameters.

Note that in problems of the mathematical programming these solution procedures are called the confidence region methods [11]. In optimal control problems the known method of sequential linearization from [2] is constructed within just the same structure.

In this paper, we study the usual optimal control problem without phase and terminal constraints. A dynamic system linearly depends on a control which is limited to a convex compact set. We approximate the functional with the help of a phase variation with a modified adjoint system. The family of admissible neighborhoods is defined with the help of the convex combination of controls. As a result, the auxiliary problem with respect to the "control, state" pair becomes bilinear and contains the parameter of the convex combination which characterizes the neighborhood of the base control. We propose to solve the auxiliary problem by the method of increments [9] which is most effective for bilinear problems. We prove the local improvement property for controls which

The work was supported by the Russian Foundation for Basic Research (project no. 05-01-00187) and the program "Universities of Russia" (project no. ur.03.01.064).