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THE COEFFICIENT CONTROL PROBLEM AND THE EXTENDED DERIVATIVE OVER A CONVEX SET

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1. Introduction

The coefficient control problems are of theoretical and practical interest. They can be obtained by reduction of the coefficient inverse problems of mathematical physics (for example, [1]) and the domain control problems [2]–[4]. The complexity of the coefficient control problems is caused, in many respects, by their possible unsolvability ([5]; [6], pp. 47–48; [7]). Though the coefficient control problems for the elliptic type equations are studied in many papers (for example, [8]–[16]), they are not thoroughly investigated within the general extremum theory, because of the peculiarity of these problems.

It is well known that the minimum point u of a smooth functional I on a convex subset U of a Banach space V satisfies the variational inequality $\langle I'(u), v-u\rangle \geq 0$ for all $v \in U$, where $I'(u)$ is the Gateaux derivative of the functional I at the point u and $\langle \lambda, h\rangle$ is the value of a linear continuous functional λ at a point h. Here the calculation of the derivative meets two principle difficulties. First of all, one has to find the value $I(u+\sigma h)$, where σ is a certain number and the direction h is an arbitrary element of the space V. The peculiarity of the problems with a control in the main part of the operator is the fact that the state equation is solvable only on a certain part of the space of controls. The unsolvability of the equation for an arbitrary control $u+\sigma h$ prevents from finding the derivative. In order to overcome this difficulty, we propose to replace in the variational inequality the Gateaux derivative with a weaker derivative over a convex set.

The second difficulty which occurs when one calculates the derivative of the minimizing functional is connected with its dependence on the state function of the system which, in turn, depends on the control. Calculating the derivative of a functional, according to the theorem on indirect differentiation in Banach spaces, implies the Gateaux differentiability of the state function with respect to the control; however, in our case, generally speaking, this dependence is not differentiable. In order to overcome this difficulty, one could have used the weaker extended operator derivative [15], [17], [18]. However, if the state equation is solvable only on a certain part of the space of controls, it is impossible to find the extended derivative of the dependence of the equation solution and the control. One can obtain the desired result with the help of the combined extended derivative over a convex set.

2. Problem definition

In an open bounded n-dimensional domain Ω consider the equation

$$-\operatorname{div}(v\nabla y) + |y|^{\rho} y = f \tag{1}$$