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AN APPROXIMATE DUAL TYPE METHOD FOR SYSTEMS OF VARIATIONAL INEQUALITIES

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1. Introduction

Variational inequalities are now a rather popular and efficient tool for investigation and solution of various equilibrium type problems (see, e.g., [1]–[3]). By definition, a variational inequality implies finding a point $z^* \in Z$ such that

$$\langle Q(z^*), z - z^* \rangle \geq 0 \quad \forall z \in Z,$$

where Z is some convex set in a normed space E, $Q : Z \to E^*$ is a given mapping (here and below E^* denotes the conjugate space for E). In [4]–[6], we consider an extended primal-dual system of variational inequalities and show that many economic equilibrium models can be reduced to this system. A solution of the system is defined as a pair of points $(x^*, y^*) \in X \times Y$ such that

$$\langle T(x^*), x - x^* \rangle + \langle y^*, H(x) - H(x^*) \rangle \geq 0 \quad \forall x \in X, \tag{1}$$

$$\langle B(y^*) - H(x^*), y - y^* \rangle \geq 0 \quad \forall y \in Y, \tag{2}$$

where X and Y are nonempty, convex, and closed subsets in R^n and

$$R^m_+ = \{y \in R^m \mid y_i \geq 0 \ i = 1, \dots, m\},$$

respectively; $T : X \to R^n$ and $B : R^m_+ \to R^m$ are given continuous mappings; $H : X \to R^m$ is a continuous mapping, whose components $H_i : X \to R$, $i = 1, \dots, m$ are convex functions. System (1), (2) also extends the optimality conditions in the form of a saddle point of the Lagrange function [4]–[6], which explains its name. In addition, in [7], we show that one can formulate network equilibrium models and several implicit optimization problems in form (1), (2). In [4]–[7], we propose to solve system (1), (2) by iterative dual type methods, whose convergence is guaranteed by the strong monotonicity properties of the mappings T and B and the Lipschitz continuity of the mapping H. Observe that one can replace the strong monotonicity conditions with those of the usual monotonicity, incorporating a suitable proximal method (see [6]) in the iterative scheme, then the auxiliary systems in this method acquire the desired properties.

However, all dual methods proposed before require the exact solution of auxiliary problems in the variables x and y, which may cause certain difficulties during their implementation, especially in the case of nonlinear mappings T and B. For this reason, the construction of approximate dual methods which can be implemented in the nonlinear case is the main goal of this paper. We also prove their convergence under the assumptions on the mappings T and B which are weaker than those done in [4]–[7] for similar exact methods. So one can directly use the proposed methods for a much more general class of applied problems.

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