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REPRESENTATION OF A SHEAF OF ADMISSIBLE TRAJECTORIES IN A LINEAR CONTROL PROBLEM WITH AN IMPULSE CONSTRAINT

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In this paper, we study one class of linear pulse-controlled systems with a discontinuity in control coefficients. Among the constraints imposed on the control we distinguish the impulse and moment components. The latter defines a condition for the control integrand. We weaken this constraint and establish the stability properties of the sheaf of trajectories under certain assumptions about the stepwise behavior of functions which generate the integrand.

1. Introduction

It is well known that under impulse constraints on the control, the system can show a certain effect which has the sense of the product of a discontinuous function and a generalized one. In this paper, following [1], we consider the problem, where this effect is connected with the discontinuity in dependences of the control coefficients (we restrict ourselves to the case of a scalar control). In addition, we assume that the control satisfies some moment and, in general, nonconvex constraints. We consider the question about the sheaf stability, weakening the mentioned moment constraints. Here we understand the stability as a topological property and, in the strict sense, relate to the closure of a sheaf in the topology of the pointwise convergence. In this paper, we formulate sufficient conditions for this stability with two formalizations of the impulse component of constraints. These conditions are based on the propositions of [1] connected with the extension of control problems in the class of finitely additive (f. a.) measures. In connection with other approaches to the construction of generalized modes of the pulse control, note [2], [3]. On the whole, the use of generalized pulse control constructions in linear systems is connected with the N.N. Krasovskii general approach [4]. The latter is a base of many investigations and future prospects in this realm. Along with [2], [3], note monographs [5]–[7].

Following [1], consider the generalized linear controlled system, using f. a. measures as controls. Here the system has another role (in comparison to [1]). Namely, using it, we establish (under certain conditions) the above property of the "topological" stability of the closure of a sheaf of trajectories, weakening the moment constraints. These conditions have been used for a long time [8]–[10], but in this case we observe some features connected with the topological framing of the space of trajectories. Note that without these conditions (the stepwise behavior of functions in correlations which define the constraints) the mentioned property which has the sense of stability, possibly, does not hold (see the relevant examples in [8]–[10] and cite25, pp. 7, 8; [10], Chap. 1).

2. Problem definition

Consider a linear controlled system on a finite time interval. The choice of the control programs is subject to impulse constraints. The latter are formalized in two specific variants: the constraint