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COMPLEXITY OF THE ALLOCATION PROBLEM ON A LINE WITH CONSTRAINTS ON MINIMAL DISTANCES

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We consider the allocation problem for interconnected rectangular objects on a line with the minimal total cost of correlations under constraints on the minimal distances. We prove that this problem is NP-complete, if the structure of correlations between the objects is either a root tree, or a graph of the series-parallel type and is polynomially resolvable for a chain.

1. Problem definition

Let n be the number of allocated objects, let $V = \{1, \dots, n\}$ be the set of their numbers. An object is a rectangle, whose dimensions are $a_i \times h_i$, $i \in V$. A correlation goes vertically from the objects centers up to the allocation line and then it does along the indicated line. The length of the vertical component of a correlation for objects i and j equals $h_i/2 + h_j/2$, $i, j \in V$, $i \neq j$. The initial minimally admissible distances are defined as those between the nearest points of the objects, however, one they can heed the values a_i, $i \in V$ there. The problem is reduced to the allocation of point objects, i. e., the projections of the geometric centers of rectangles on the line. We denote the minimally admissible distance between objects i and j by r_{ij}; $R = (r_{ij})$, $r_{ij} = r_{ji}$, $r_{ii} = 0$, $i, j \in V$, is the matrix of minimally admissible distances. The structure of correlations between the objects is defined by the graph $\Gamma = (V, E)$. An arc $(i, j) \in E$, if a correlation between objects i and j exists such that its specific cost $C_{ij} > 0$, $C_{ij} = C_{ji}$. If for the connected objects the order of mutual disposition on the line is given, for example, the i-th object has to be located to the left of the j-th one, then (i, j) is an arc.

Let the coordinate axis be directed along the allocation line and let x_i be the coordinate of the center of object i, $i \in V$, then the model of the formulated problem has the form

$$f(x_1, x_2, \dots, x_n) = \sum_{(i,j)\in E} C_{ij}|x_i - x_j| \to \min, \tag{1.1}$$

$$|x_i - x_j| \geq r_{ij}, \quad i, j \in V, \quad i < j. \tag{1.2}$$

Problem (1.1), (1.2) is NP-complete, because if $r_{ij} = 1$, $i, j \in V$, $i \neq j$; $C_{ij} = 1$, $(i, j) \in E$, and Γ is an arbitrary undirected graph, then it is equivalent to the NP-complete problem of the optimal linear ordering ([1], p. 250).

2. The complexity of problem (1.1), (1.2)

Consider the following constraints on the elements of the matrix R:

a) $r_{ij} = (a_i + a_j)/2$, $i, j \in V$, $i \neq j$ (the non-intersection conditions);

b) $r_{ij} + r_{jk} \geq r_{ik}$, $i, j, k \in V$, $i \neq j \neq k$ (the metric problem);

c) r_{ij} are arbitrary, $i, j \in V$ (the nonmetric problem).

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