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QUASI-STABILITY MEASURE OF THE l_1-METRIC FOR A VECTOR COMBINATORIAL PROBLEM WITH A PARAMETRIC OPTIMALITY PRINCIPLE

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1. Introduction

Traditionally, one considers stability of optimization problems (both scalar and vector ones) as a continuous dependence of their solutions on the problem parameters. The most general approaches to the investigation of stability of optimization problems are based on the analysis of properties of multi-valued (point-to-set) mappings which define the optimality principle (the choice function) [1]–[4].

The traditional methods of mathematical analysis are insufficient for the investigation of stability of discrete optimization problems. This fact is mainly caused by the complexity of discrete models, whose behavior is often unpredictable even under small perturbations of the initial data [4], [5]. At the same time, rejecting the usual tools of the general topology with respect to the space of isolated points significantly simplifies the definition of the stability problem. There exist various kinds of stability of discrete optimization problems (e. g., [4]–[9]). In the wide sense, stability of a discrete problem implies the existence of a neighborhood of the initial data in the space of problem parameters such that any "perturbed" problem, whose parameters belong to this neighborhood, possesses a certain prescribed invariance property. For example, the upper (lower) semicontinuity of the optimal mapping in the Hausdorff sense is equivalent to the absence of new optimal solutions (i. e., the invariance of the initial ones) under "small" perturbations of problem parameters. This raises the concepts of stability [4]–[8] and quasi-stability [6]–[8], [10], [11] of discrete optimization problems.

In this paper, parameterizing the optimality principles (from Pareto to lexicographic ones), we obtain a unified formula for the quasi-stability radius in the l_1-metric for various kinds of n-criteria linear trajectory problems. The latter arise under an arbitrary partition of partial criteria into groups, so that the Paretian optimality criterion is defined within each group and the lexicographic one is defined for different groups. Several qualitative type results are obtained as conclusions of this formula.

Observe that the similar formulae of the stability and quasi-stability radii in the metric l_∞ are obtained in [12]–[16] for other kinds of parameterization of the optimality principles in the vector trajectory and game-theoretic problems ("from Condorcet to Pareto", "from Pareto to Slater", "from Pareto to Nash").

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