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THE GOURSAT PROBLEM FOR A THREE-DIMENSIONAL EQUATION WITH HIGHER DERIVATIVE

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In the parallelepiped $D = \{x_0 < x < x_1,\ y_0 < y < y_1,\ z_0 < z < z_1\}$ we consider the equation

$$L(u) \equiv \sum_{\alpha,\beta=0}^{2} \sum_{\gamma=0}^{1} a_{\alpha\beta\gamma}(x,y,z) \frac{\partial^{\alpha+\beta+\gamma} u}{\partial x^\alpha \partial y^\beta \partial z^\gamma} = F(x,y,z). \tag{1}$$

Its plane analog, i. e., when $a_{\alpha\beta 1} \equiv 0$ for all α, β (z can be considered as a parameter or assumed to be absent), contains the Boussinesq–Lave equation $u_{xxyy} - u_{xx} + u_{yy} = 0$ known in the theory of oscillations (see [1], formula (20)) and the Aller equation $u_y = (au_x + bu_{xy})_y$, which describes the process of filtration in the moisture absorption by roots of plants (see [2], p. 261). The general two-dimensional equation with the higher derivative u_{xxy} was considered in [3]–[9]. In (1) one can find also the case of the equation with the higher derivative u_{xyz}, which was studied in [10]–[12]. Equations with such a derivative are used in the simulation of vibration and play an essential role in the theory of approximation and mappings (see [13], p. 109). To equations of that kind the problem of integral representation of a transformation of ordinary linear differential operators to other ones can be reduced (see [14]). Thus, we can consider (1) as a generalization of a series of equations investigated earlier.

We will assume that $a_{221} \equiv 1$, the smoothness of other coefficients is determined by the inclusions

$$a_{\alpha\beta\gamma} \in C^{\alpha+\beta+\gamma}(\overline{D}), \quad F \in C^{2+2+1}(\overline{D}), \tag{2}$$

where $C^{\alpha+\beta+\gamma}$ is the class of functions continuous in $\overline{D}$ together with their derivatives $\dfrac{\partial^{r+s+t}}{\partial x^r \partial y^s \partial z^t}$ ($r = 0, \dots, \alpha$; $s = 0, \dots, \beta$; $t = 0, \dots, \gamma$).

1. Let X, Y, and Z be faces of D for $x = x_0$, $y = y_0$, and $z = z_0$, respectively.

(Goursat) Problem. Find in D a solution of equation (1) of class C^{2+2+1}, which satisfies the boundary conditions

$$\begin{gathered} u|_X = \varphi(y,z), \quad u|_Y = \psi(x,z), \quad u|_Z = \theta(x,y), \\ u_x|_X = \varphi_1(y,z), \quad u_y|_Y = \psi_1(x,z), \\ \theta \in C^{2+2}(\overline{Z}), \quad \varphi, \varphi_1 \in C^{2+1}(\overline{X}), \quad \psi, \psi_1 \in C^{2+1}(\overline{Y}). \end{gathered} \tag{3}$$

On the edges of D the following conditions are assumed to be fulfilled

$$\begin{gathered} \varphi(y_0, z) = \psi(x_0, z), \quad \varphi(y, z_0) = \theta(x_0, y), \quad \psi(x, z_0) = \theta(x, y_0), \\ \varphi_1(y_0, z) = \psi_x(x_0, z), \quad \psi_1(x_0, z) = \varphi_y(y_0, z), \end{gathered}$$

and, in addition, the proper coordinated values are continuously differentiable.