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CHOICE OF PARAMETERS IN THE METHOD OF REGULARIZATION OF L-PSEUDOINVERSION

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In [1] a two-parametric method for regularization of the problem of L-pseudoinversion was suggested, whose domain of application is wider than that of the one-parametric regularization method completely investigated in [2]. In [1] both the convergence and stability of the method were established, in [3] some criteria for the choice of regularization parameters were justified. In this article we suggest and justify criteria for consecutive choice of both the parameters of regularization.

1. Introduction

Let closed linear operators $A : X \to Y$, $B : X \to Z$ be given, where X, Y, and Z are Hilbert spaces, with the domains of definition $D(A)$ and $D(B)$, respectively. Suppose that their common domain of definition $D = D(A) \cap D(B)$ is everywhere dense in X. Let, in addition, the two elements $y \in Y$, $z \in Z$ be given.

We define the sets

$$X_1 = \{x \in D \mid \|Bx - z\| = \inf_{u \in D} \|Bu - z\| = \mu_B\},$$
$$X_2 = \{x \in X_1 \mid \|Ax - y\| = \inf_{u \in X_1} \|Au - y\| = \mu_A\}$$

and pose the following problem: Find an element of minimal norm x^* in the set X_2. In the mathematical literature this problem is called the problem of L-pseudoinversion. For the sake of brevity here we will call it the main problem and also assume that both the operators A and B satisfy the following generalized complementabilty condition:

$$\exists\, \gamma > 0 : \|Ax\|^2 + \|Bx\|^2 \geq \gamma^2 \|x\|^2 \quad \forall\, x \in D^{\perp}, \tag{1}$$

where $D^{\perp} = D \cap (N(A) \cap N(B))^{\perp}$, while $N(A)$ and $N(B)$ are kernels of the operators A and B, respectively.

In [2] and [4], for the main problem condition (1) was supposed to be fulfilled for all $x \in D$. Hence, in particular, the relation follows $N(A) \cap N(B) = \{0\}$, and therefore the set X_2 is at most countable. One can easily see (e. g., [5]) that the solvability of the main problem is equivalent to the non-emptiness of the set X_1, because it is closed and convex and the latter is equivalent to the condition $z \in D(B^+) = R(B) \oplus R(B)^{\perp}$. It is also known that the solution of the main problem x^* lies in $D^{\perp}$.

In [3], [5] the approximation of the main problem was based on the regularizing functional

$$\Phi_{r\alpha}[x] = r\|Bx - z\|^2 + \|Ax - y\|^2 + \alpha\|x\|^2, \tag{2}$$