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COMPLETELY IDEMPOTENT PSEUDOCONNECTIONS ON SEMI-RIEMANNIAN AND PSEUDO-RIEMANNIAN SPACES AND CONCIRCULAR FIELDS

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Introduction

This article is a sequel to the series of author's papers devoted to the theory of pseudoconnections (see [1]–[3]). Section 1 is an introduction, we cite here some facts from the theory of completely idempotent pseudoconnections. In Section 2, applying the apparatus of this theory, we construct a canonical connection on a pseudo-Riemannian almost product space and expose Naveira's classification (see, e. g., [4]) in terms of this connection. In Sections 3 and 4 we study special classes of concircular fields on a semi-Riemannian space (a manifold endowed with a degenerate metric). All objects are considered in the local form, the functions involved are assumed to belong to sufficiently high differentiability class (we call them smooth).

1. Idempotent pseudoconnections and semi-Riemannian almost product spaces

1. Let M be a smooth n-dimensional manifold, $\mathcal{F}(M)$ the ring of smooth functions on M, $\mathfrak{X}(M)$ the Lie algebra of smooth vector fields on M, and X, Y, Z, W arbitrary smooth vector fields on M.

Definition 1 (see [5]). *A linear pseudoconnection* on M is a pair of operators $(h;\nabla)$, where $\nabla : \mathfrak{X}(M) \times \mathfrak{X}(M) \longrightarrow \mathfrak{X}(M)$ and h is a tensor field of type $(1;1)$ on M, which satisfy the conditions

$$\nabla_X(fY+Z) = f\nabla_X Y + X(f)hY + \nabla_X Z,$$
$$\nabla_{fX+Y}Z = f\nabla_X Z + \nabla_Y Z \quad \forall X,Y \in \mathfrak{X}(M), \quad f \in \mathcal{F}(M).$$

Definition 2. *A linear quasiconnection* on M is a pair of operators $(h;Q)$, where $Q : \mathfrak{X}(M) \times \mathfrak{X}(M) \longrightarrow \mathfrak{X}(M)$ and h is a tensor field of type $(1;1)$ on M, which satisfy the conditions

$$Q_X(fY+Z) = fQ_X Y + hX(f)Y + Q_X Z,$$
$$Q_{fX+Y}Z = fQ_X Z + Q_Y Z \quad \forall X,Y \in \mathfrak{X}(M), \quad f \in \mathcal{F}(M).$$

A linear pseudoconnection (quasiconnection) on M with $h = \mathrm{id}$ is a linear connection.

Definition 3 (see [5]). The tensors

$$S(X,Y) = \nabla_X Y - \nabla_Y X - h[X,Y], \tag{1}$$
$$R(X,Y)Z = \nabla_X\nabla_Y Z - \nabla_Y\nabla_X X - \nabla_{[X,Y]}Z, \tag{2}$$
$$\mathrm{Ric}(X,Y) = \mathrm{tr}\, R(X,Y)$$