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THREE-WEBS DETERMINED BY A LINEAR DIFFERENTIAL EQUATION

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Introduction. In [1], [2] the systematic investigation of classes of three-webs defined by relations on relative curvature invariants was started. It was proved that one of the covariant derivatives of curvature of a three-web W vanishes if and only if two families of W satisfy the generalized Abel equations

$$\frac{dy}{dx} = -ay^n + b(x)y^{n-2} + \cdots + c(x).$$

In this article we consider the problem which is in a certain sense inverse to the above one. Namely, we pose a problem on investigation of three-webs defined by special differential equations, and characterize analytically three-webs determined by an arbitrary first order differential equation. We demonstrate that the class of these webs is characterized by vanishing of relative invariants.

Every linear differential equation of the form

$$y' = f(x, y) \tag{1}$$

determines a three-web W consisting of the three families of curves λ_α,

$$\lambda_1 : x = \text{const}, \quad \lambda_2 : y = \text{const}, \quad \lambda_3 : F(x, y) = \text{const},$$

where λ_3 consists of the integral curves of (1). Conversely, each curvilinear three-web W is equivalent to a three-web $\widetilde{W}$ consisting of the three families λ_α, where, in addition, the leaves of the third foliation of $\widetilde{W}$ are integral curves of the ordinary differential equation $F_x dx + F_y dy = 0$. This equation is determined not uniquely but up to transformations of the form

$$x = \alpha(\widetilde{x}), \quad y = \alpha(\widetilde{y}), \tag{2}$$

which transform the Cartesian web $x = \text{const}$, $y = \text{const}$ into itself. It is conventional to consider a three-web given by an equation $z = F(x, y)$ up to isotopic transformations, i. e., up to local diffeomorphisms of the form $x = \alpha(\widetilde{x})$, $y = \alpha(\widetilde{y})$, $z = \alpha(\widetilde{z})$. In view of the above correspondence between three-webs and differential equations of the form (1), one can consider these equations also up to an isotopy of the form (2). To be more precise, we can say that the web theory under consideration catches the properties of differential equations which are invariant under isotopic transformations (2). This approach makes it possible to classify the ordinary differential equations up to the isotopy through the use of the differential geometric invariants of appropriate three-webs. On the other hand, we can interpret the properties of solutions of differential equations in terms of the properties of the corresponding three-webs.

In this article we consider the simplest case, i. e., the equation

$$y' + yf(x) = g(x), \tag{3}$$

and characterize the corresponding class of three-webs in an invariant way.