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# PROBLEM OF HEAT-CONVECTION OF THE INCOMPRESSIBLE VICSOELASTIC KELVIN–VOIGT FLUID OF NONZERO ORDER

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The system of equations

$$\begin{aligned} &(1-\lambda\nabla^2)\mathbf{v}_t = \nu\nabla^2\mathbf{v} - (\mathbf{v}\cdot\nabla)\mathbf{v} + \sum_{l=1}^{k}\beta_l\nabla^2\mathbf{w}_l - g\mathbf{q}\theta - \mathbf{p} + \mathbf{f}, \\ &0 = \nabla(\nabla\cdot\mathbf{v}), \\ &\frac{\partial\mathbf{w}_l}{\partial t} = \mathbf{v} + \alpha_l\mathbf{w}_l, \quad \alpha_l\in\mathbb{R}_-, \quad l=\overline{1,k}, \\ &\theta_t = \varkappa\nabla^2\theta - \mathbf{v}\cdot\nabla\theta + \mathbf{v}\cdot\mathbf{q} \end{aligned} \tag{1}$$

simulates the evolution of the velocity $\mathbf{v} = (v_1,\dots,v_n)$, $v_i = v_i(x,t)$, of the pressure gradient $\mathbf{p} = (p_1,\dots,p_n)$, $p_i = p_i(x,t)$, and of the temperature $\theta = \theta(x,t)$ of the incompressible viscoelastic Kelvin–Voigt fluid of order $k > 0$ (see [1], [2]). The parameters $\lambda\in\mathbb{R}$, $\nu\in\mathbb{R}_+$, and $\varkappa\in\mathbb{R}_+$ characterize the elasticity, viscosity, and heat conductivity of the fluid, respectively; $g\in\mathbb{R}_+$ is the gravitational acceleration; the vector $\mathbf{q} = (0,\dots,0,1)$ is the unit vector in $\mathbb{R}^n$. The parameters $\beta_l\in\mathbb{R}_+$, $l=\overline{1,k}$, define the time of retardation (delay) of the pressure. The free term $\mathbf{f} = (f_1,\dots,f_n)$, $f_i = f_i(x,t)$, corresponds to the outer action on the fluid.

We investigate the solvability of the first initial boundary value problem

$$\begin{aligned} &\mathbf{v}(x,0) = \mathbf{v}_0(x), \quad \mathbf{w}_l(x,0) = \mathbf{w}_{l_0}(x), \quad \theta(x,0) = \theta_0(x) \ \ \forall x\in\Omega; \\ &\mathbf{v}(x,t) = 0, \quad \mathbf{w}_l(x,t) = 0, \quad \theta(x,t) = 0 \ \ \forall (x,t)\in\partial\Omega\times\mathbb{R}_+, \quad l=\overline{1,k}, \end{aligned} \tag{2}$$

for system (1). Here $\Omega\subset\mathbb{R}^n$, $n = 2,3,4$, is a bounded domain with the boundary $\partial\Omega$ of class $C^\infty$.

Earlier problem (1), (2) in a particular case ($k=0$, $f=f(x)$) was studied by G.A. Sviridyuk in [3]. For the incompressible viscoelastic Kelvin–Voigt fluid of order $k>0$ with a nonstationary free term $f(x,t)$ this problem is considered for the first time.

In this article we establish the local unique solvability of problem (1), (2). This problem will be considered in the framework of the Sobolev type equations on the basis of the notion of a relatively $p$-sectorial operator and the semigroup approach suggested in [4].

## 1. Abstract problem

Let $\mathcal{U}$ and $\mathcal{F}$ be Banach spaces, the operator $L\in\mathcal{L}(\mathcal{U};\mathcal{F})$, i.e., is linear and continuous, $\ker L\neq\{0\}$; the operator $M:\operatorname{dom}M\to\mathcal{F}$ is linear, closed and densely defined in $\mathcal{U}$. We denote $\mathcal{U}_M = \{u\in\operatorname{dom}M : \|u\| = \|Mu\|_{\mathcal{F}} + \|u\|_{\mathcal{U}}\}$. Let the function $F$ belong to $\mathcal{C}^\infty(\mathcal{U}_M;\mathcal{F})$, the function $f$ to $\mathcal{C}^\infty(\overline{\mathbb{R}}_+;\mathcal{F})$.

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