Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 45, No. 11, pp.32–43, 2001 *UDC 517.968:519.6*

SPLINE APPROXIMATIONS OF SOLUTION OF SINGULAR INTEGRO-DIFFERENTIAL EQUATION

E.N. Samoĭlova

Introduction

In this article for the equation

$$A\varphi \equiv \varphi'(t) + a(t)\varphi(t) + \frac{b(t)}{\pi}\int_{-1}^{1}\frac{\varphi(\tau)d\tau}{\tau - t} = f(t), \quad -1 \le t \le 1, \tag{0.1}$$

with the initial condition

$$\varphi(-1) = 0 \tag{0.2}$$

the computational schemes of the spline-collocation and spline-subdomain methods are given with the theoretical substantiation of the methods (see [1], Chap. 14; [2], Chap. 1).

1. Computational scheme of the spline-collocation method

We give the computational scheme of the spline-collocation method for problem (0.1)–(0.2) for $a(t), b(t), f(t) \in C[-1,1]$, where $C[-1,1] \equiv C$ is the space of continuous on $[-1,1]$ functions with the ordinary norm.

We introduce the grids of nodes

$$t_k = -1 + \frac{2k}{n}, \quad k = \overline{0,n}, \quad n \in N, \tag{1.1}$$

$$\overline{t}_{k-1} = \frac{t_{j-1} + t_j}{2} = -1 + \frac{2j-1}{2n}, \quad j = \overline{1,n}, \quad n \in N, \tag{1.2}$$

where N is the set of natural numbers. The approximate solution of problem (0.1)–(0.2) will be sought in the form of the spline

$$\varphi_n(t) = \sum_{k=0}^{n} \alpha_k s_k(t), \quad n \in N, \tag{1.3}$$

where

$$s_k(t) = \begin{cases} 0, & t \le t_{k-1}; \\ \frac{t - t_{k-1}}{t_k - t_{k-1}}, & t_{k-1} \le t \le t_k; \\ \frac{t_{k+1} - t}{t_{k+1} - t_k}, & t_k \le t \le t_{k+1}; \\ 0, & t \ge t_{k+1}, \end{cases} \tag{1.4}$$