Russian Mathematics (Iz. VUZ)
Vol. 45, No. 11, pp.19–31, 2001

Izvestiya VUZ. Matematika
UDC 517.95

THE GELLERSTEDT PROBLEM FOR SYSTEMS OF EQUATIONS OF MIXED TYPE

K.B. Sabitov and R.G. Idrisov

1. Statement of problem

Let us consider the system

$$LU \equiv K(y)\mathbf{U}_{xx} + \mathbf{U}_{yy} + A(x,y)\mathbf{U}_x + B(x,y)\mathbf{U}_y + C(x,y)\mathbf{U} = 0, \tag{1}$$

where $yK(y) > 0$ for $y \neq 0$, $K(y)$, $A(x,y)$, $B(x,y)$ are given numerical functions, $C(x,y) = (C_{ik}(x,y))$, $i,k = \overline{1,n}$, is a square matrix, $\mathbf{U} = (u_1, u_2, \dots, u_n)$, $n \geq 2$, in the domain D bounded by a simple Jordan curve Γ lying in the halfplane $y > 0$ with ends at points $A_1(a_1, 0)$ and $A_2(a_2, 0)$, $a_1 < a_2$, the characteristics A_1C_1, C_1E, EC_2, C_2A_2 of system (1) for $y < 0$, where $E(e, 0)$, $a_1 < e < a_2$, $C_1(\frac{a_1+e}{2}, y_{c_1})$, $y_{c_1} < 0$ and $C_2(\frac{a_2+e}{2}, y_{c_2})$, $y_{c_2} < 0$.

Denote $D_+ = D \cap \{y > 0\}$, $D_1 = D \cap \{y < 0 \wedge x < e\}$ and $D_2 = D \cap \{y < 0 \wedge x > e\}$. In what follows we assume

$$K(y) \in C(\overline{D}_+) \wedge C(\overline{D_i}) \wedge C^2(\overline{D_i} \setminus \overline{EA_i}), \quad C_{jk}(x,y) \in C(\overline{D}_+) \wedge C(\overline{D_i}), \quad j,k = \overline{1,n},$$
$$A(x,y), B(x,y) \in C^1(\overline{D}_+) \wedge C(\overline{D_i}) \wedge C^1(\overline{D_i} \setminus \overline{EA_i}), \quad i = 1,2.$$

For system (1) in the domain D we consider the Gellerstedt problem (Problem G).

Problem G. Find a function $\mathbf{U}(x,y)$ which satisfies the conditions

$$\mathbf{U}(x,y) \in C(\overline{D}) \wedge C^1(D) \wedge C^2(D_+ \cup D_1 \cup D_2), \tag{2}$$
$$L\mathbf{U}(x,y) \equiv 0, \quad (x,y) \in D_+ \cup D_1 \cup D_2, \tag{3}$$
$$\mathbf{U}(x,y) = \Phi(x,y), \quad (x,y) \in \overline{\Gamma}, \tag{4}$$
$$\mathbf{U}(x,y) = \Psi(x,y), \quad (x,y) \in \overline{A_1C_1} \cup \overline{A_2C_2}, \tag{5}$$

where Φ and Ψ are given sufficiently smooth vector functions and $\Phi(A_1) = \Psi(A_1)$ and $\Phi(A_2) = \Psi(A_2)$.

In this article we establish the extremal properties of the module

$$|\mathbf{U}(x,y)| = \sqrt{\sum_{i=1}^{n} u_i^2(x,y)}$$

of solutions of the Gellerstedt problem for system (1) in the domains of ellipticity, hyperbolicity, and in the whole in the mixed domain D. We give also applications of these properties to the investigation of Problem G.

Supported by the Russian Foundation for Basic Research, grant no. 99-01–00934.