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STABILIZATION OF SOLUTION OF THE CAUCHY PROBLEM FOR AN ABSTRACT DIFFERENTIAL EQUATION OF THE FIRST ORDER

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In a Banach space E we consider the abstract Cauchy problem:

$$v'(t) = Av(t), \quad t > 0, \tag{1}$$

$$v(0) = v_0, \tag{2}$$

where A is a linear closed operator with the domain of definition $D(A)$ dense in E and such that, for $k > 0$, $\gamma > 0$, the following Cauchy problem is uniformly correct:

$$u''(t) + \gamma k \operatorname{cth} \gamma t u'(t) + \left(\frac{\gamma k}{2}\right)^2 u(t) = Au(t), \tag{3}$$

$$u(0) = u_0, \quad u'(0) = 0. \tag{4}$$

Following [1] (item 1.9; [2]), we denote by G_k^γ the class of these operators A and by $P_k^\gamma(t)$ the resolving operator of problem (3), (4) (the operator Legendre function (OLF)). Thus, solution of problem (3), (4) has the form $u(t) = P_k^\gamma(t)u_0$.

Parameter $\gamma > 0$ is introduced into equation (3) which we call the abstract Legendre equation in order to emphasize that, as $\gamma \to 0$, it turns into the abstract Euler–Poisson–Darboux equation:

$$u''(t) + \frac{k}{t}u'(t) = Au(t). \tag{5}$$

A number of definitive results were currently obtained in the theory of stabilization of solution of the Cauchy problem for parabolic equations. A survey of published results concerning this topic can be found in [3], [4]. These results are formulated in terms of averaging of the initial function with respect to spatial variables which are taken in some bodies bounded by level surfaces of the fundamental solution. The averagings mentioned above are solutions of the corresponding Cauchy problem of the form (5), (4) for a hyperbolic equation; this made it possible to investigate in [5]–[7] the questions of the behavior of the abstract Cauchy problem (1), (2) as $t \to \infty$ under the assumption of the uniform correctness of problem (5), (4). The stabilization criteria obtained were formulated in terms of the resolving operator of problem (5), (4), which was called by the author "the operator Bessel function" (OBF), represent abstract analogs of the known results from the theory of parabolic equations.

In contrast to [8], [9], where the stabilization conditions for solution of problem (1), (2) were formulated in terms of a uniformly bounded semigroup generated by the operator A, in [5]–[7] the integral representation of the solution of the Cauchy problem is obtained namely due to restriction of the class of operators A from the set of semigroup generators to the set of generators of OBF. This representation is put into the basis of the stabilization criterion and thus allows us to consider problem (1), (2) also with the generator of the semigroup which grows as $t \to \infty$.