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VARIATIONAL MAXIMUM PRINCIPLE IN CONTROLLED SYSTEMS OF ONE-DIMENSIONAL HYPERBOLIC EQUATIONS

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Introduction

The variational maximum principle was first obtained as an autonomous result by V.A. Srochko (see [1]) for hyperbolic systems of canonical form and Goursat–Darboux form. The simplicity and factual equivalence of these systems can be explained by similar construction of the families of their characteristics: Each characteristic of one family is orthogonal to any characteristic of another family. Later the variational maximum principle was successfully extended to more general hyperbolic systems (see [2]), where curvilinear characteristics of an arbitrary quantity of families appear. However, in [2]–[4] they essentially used the initial form of writing of hyperbolic system in the Riemann invariants, which allowed to restrict rather easily the differential operator to the characteristics and obtain necessary estimates for the increment of the trajectory via the parameters of variations of control. Further attempts to generalize the given result to systems of multidimensional hyperbolic equations revealed the necessity to apply a principally different technique, since, in the general case, the hyperbolic systems cannot be reduced to an invariant form in the ordinary sense of this term.

In the present article, as before in [2]–[4], we obtain the variational maximum principle for one-dimensional hyperbolic systems, but these systems are written in a general, non-invariant form. We may cite the following circumstances for justification of this "intermediary" objective. First of all, the preliminary investigation of the one-dimensional systems of a general form allows to prepare and probe rather easily many constructions, notions, assertions, which change insignificantly in being transferred to multidimensional systems, at least from a formal standpoint. Secondly, in most applications the systems are usually written in a nontrivial form. Their reduction to an invariant form requires a recalculation by means of a linear change of variables of the system of differential equations, the initial and boundary conditions, as well as the goal functional. Clearly, it is more convenient to use the optimality condition formulated in terms of the initial statement of problem. Finally, third, an important moment is the possibility to check the new result by comparison with those already possessed.

1. Statement of the optimal control problem

In the plane of independent variables (s,t) we consider the rectangle $\mathrm{P} = S \times T$, $S = [s_0, s_1]$, $T = [t_0, t_1]$. Let Ω, G, G_0, G_1, $\mathcal{D}_0$, $\mathcal{D}_1$ be its complete, lateral, left, right, upper, and lower boundaries, respectively, $\Omega = G \cup \mathcal{D}_0 \cup \mathcal{D}_1$, $G = G_0 \cup G_1$. Inside the rectangle P we fix a vertical interval $\overline{G} = \{(s,t) : s = \overline{s},\ t \in T\}$ by choosing $\overline{s} \in (s_0, s_1)$.