Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 43, No. 12, pp. 67–69, 1999 *UDC 517.977*

ON A WAY TO CONSTRUCT THE PARAMETRIC SYNTHESIS FOR A LINEAR-QUADRATIC PROBLEM OF OPTIMAL CONTROL

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In the present article we suggest a way for construction of optimal control for a linear-quadratic problem of optimal control with arbitrary parameters or "perturbations" as a function of independent variable and parameters. Such a control, preserving the optimality property for any perturbations of the problem, can be related to the class of robust controls.

1. Statement of the problem

Let us consider a problem of minimization of the quadratic integral functional

$$J(u) = \int_T [\langle g(t), x(t)\rangle + \langle d(t), u(t)\rangle + \tfrac{1}{2}\langle G(t)x(t), x(t)\rangle + \tfrac{1}{2}\langle D(t)u(t), u(t)\rangle]dt, \tag{1.1}$$

defined on solutions of the linear system of ordinary differential equations

$$\dot{x} = A(t)x + B(t)u + C(t)p + f(t) \tag{1.2}$$

with the conditions

$$x(t_0) = q, \quad x(t_1) = r \tag{1.3}$$

at the end points of the segment $T = [t_0, t_1]$. Here $x = x(t)$, $x(t) \in E^n$, is a state of the process, $u = u(t)$, $u(t) \in E^r$, is the control, $\langle \cdot, \cdot \rangle$ is a symbol for the scalar product of vectors in E^n or in E^r, $A(t)$, $B(t)$, $C(t)$, $G(t)$, $D(t)$, $f(t)$, $g(t)$, and $d(t)$ are given continuous matrix functions of dimensions $(n \times n)$, $(n \times r)$, $(n \times k)$, $(n \times n)$, $(r \times r)$, $(n \times 1)$, $(n \times 1)$, $(r \times 1)$, respectively. The matrix $G(t)$ is symmetric and non-negative definite, while the matrix $D(t)$ is symmetric positive definite at any $t \in T$. The vectors $p \in E^k$, $q \in E^n$, $r \in E^n$ are arbitrary parameters, $\alpha = (p, q, r)$, i.e., α stands for arbitrary perturbations of the problem. We search the optimal control u^* in the form of the function $u^* = u^*(t, \alpha)$.

2. Solving method

The optimal control u^* of the stated above problem satisfies the L.S. Pontryagin maximum principle for any fixed value of the parameters $\alpha = (p, q, r)$. Therefore the solution of the boundary value problem

$$\dot{x} = A(t)x + B(t)D^{-1}(t)B(t)'\psi - B(t)D^{-1}(t)d(t) + C(t)p + f(t), \quad \dot{\psi} = G(t)x - A(t)'\psi + g(t), \tag{2.1}$$

$$x(t_0) = q, \quad x(t_1) = r \tag{2.2}$$

Supported by Russian Foundation for Basic Research and Federal Objective Program "Integration", grant A 0037.