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ON A CLASS OF D-GAP FUNCTIONS FOR MIXED VARIATIONAL INEQUALITIES

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1. Introduction

Let U be a nonempty closed convex set in the n-dimensional Euclidean space R^n, $G : R^n \to R^n$ a continuously differentiable mapping, $f : R^n \to R$ a convex and continuous function. The mixed variational inequality problem is the problem of finding $u^* \in U$ such that

$$\langle G(u^*), u - u^* \rangle + f(u) - f(u^*) \geq 0 \quad \forall u \in U. \tag{1}$$

In its general form, problem (1) was considered in [1] and afterwards studied by many authors (see, e. g., [2]). It has many applications in the mathematical physics, and in the case where $f \equiv 0$, it corresponds to the well-known variational inequality problem. One of the most popular approaches to solve the variational inequality problem is to convert it into an optimization problem with respect to some artificial merit function. Such differentiable merit functions were first proposed in [3], [4]. Namely, they convert the variational inequality into a constrained differentiable optimization problem. Recently, a new merit function for the usual variational inequality, which allows to convert it into an unconstrained differentiable optimization problem, was introduced in [5]. In [6], where the corresponding unconstrained differentiable optimization algorithm were proposed, such functions were termed D-gap functions.

For more general mixed variational inequalities, various classes of merit functions, which are based on those in [3], [4], were proposed in [7]. However, the initial problem then reduces to a constrained optimization problem which involves, in general, a nondifferentiable cost function. In the present article we introduce a class of D-gap functions for problem (1) which enables us to convert it into an unconstrained differentiable optimization problem. We show that such a D-gap function is differentiable, so that we can apply the known rapidly convergent algorithms in order to find a solution of the initial mixed variational inequality.

2. Gap functions

First we consider the function

$$\varphi_\alpha(u) = \max_{v \in U} \Phi_\alpha(u, v), \tag{2}$$

where

$$\Phi_\alpha(u, v) = \langle G(u), u - v \rangle - 0.5\alpha \|u - v\|^2 + f(u) - f(v). \tag{3}$$

The function $\Phi_\alpha(u, \cdot)$ is strongly concave, hence, a unique solution to problem (2) exists, i. e., an element $v_\alpha(u) \in U$ such that $\Phi_\alpha(u, v_\alpha(u)) = \varphi_\alpha(u)$. Note that more general gap functions may also

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