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ALGORITHMS IN THE METHOD OF CENTERS WITH INCOMPLETE MINIMIZATION OF THE AUXILIARY FUNCTION OF MAXIMUM

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Realization of the classical method of centers (see [1], [2]) presumes the use of two infinite iterative processes, because each point x_k, $k = 1, 2, \dots$, of the main iterative sequence is sought in the result of an infinite process of unconditional minimization of a certain auxiliary function of maximum. Apparently, namely this circumstance restrains the use of the method of centers in solving problems of mathematical programming. In the parameterization of the method of centers (see [3]) it was shown that, principally, one can indicate the values of controlling parameters, with which already the point x_1 will be an approximation with a prescribed accuracy. Moreover, under certain conditions, it may be even the exact solution of the initial problem of mathematical programming. In methods with adaptation of parameter (see [4]–[6]), this feature was used in the construction of feasible algorithms. Another approach to determination of an approximate solution with the accuracy given via the functional was suggested in algorithms with two-side approximation (see [7]–[9]).

In the present article we suggest general approaches for provision of the convergence of the method of centers and determination of an approximate solution with a prescribed accuracy under incomplete minimization of the auxiliary function of maximum. We give general schemes and feasible algorithms of determination of a solution with prescribed accuracy within a finite number of steps. The infinite process of unconditional minimization is kept only as the criterion of attaining of the ε-solution. Note that in [10] an algorithm with incomplete minimization of the auxiliary function of maximum was suggested, but only with the use of the method of search of minimax in [11].

Everywhere in what follows the functions $f(x)$, $f_i(x)$, $i \in H = \{1, 2, \dots, m\}$, are definite and continuous in R_n, $D = \{x : x \in R_n,\ f_i(x) \le 0,\ i \in H\} = \{x : x \in R_n,\ g(x) \le 0\}$, where $g(x) = \max\{f_i(x),\ i \in H\}$, the set $D' = \{x : x \in R_n,\ g(x) < 0\}$ is nonempty and its closure $\overline{D'}$ coincides with D, $f^* = \inf\{f(x),\ x \in D\}$. We denote by x^* a point in $D^* = \{x : x \in D,\ f(x) = f^*\}$, $\varepsilon \ge 0$. It is required to determine

$$\inf\{f(x),\ x \in D\}. \tag{1}$$

In using the symbol inf we assume that the most lower bound of the function is attained on the mentioned set.

I. In this section we assume $F(x, y, \varepsilon) = \max\{f(x) - f(y) + \varepsilon,\ g(x)\}$, $D(y, \varepsilon) = \{x : x \in D,\ f(x) - f(y) + \varepsilon < 0\}$.

Definition 1. A point $y \in D$ will be called ε-solution of problem (1) if the inequality holds: $f(y) \le f^* + \varepsilon$.