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THE GENERAL THEORY OF STABILITY IN LINEAR PROGRAMMING

I.I. Yeryomin

1. Introduction

There is a great deal of works concerning stability problems in the mathematical programming (MP). The traditional statement of one of these problems consists of the following: If $\widetilde{f}(y)$ is the optimum function of an MP problem with a selected parameter y, then what are conditions ensuring the existence and continuity of the function $\widetilde{f}(y)$ in a neighborhood of a fixed point y_0? Usually, they formulate sufficiently rigid constraints which cannot provide a contensive analysis of the stability even in the case of linear programming (LP) problems; for example, as it concerns the question on a necessary and sufficient condition of the S-stability of the LP problem, where S is a subset of the whole system of input data. We should mention works [1]–[3] related to the stability in MP and convex programming (CP).

For the LP problem

$$\max\{(c,x) \mid Ax \le b,\ x \ge 0\} \tag{1}$$

in the capacity of y they take either the whole system of input data $[A,b,c]$, or a certain its part: b, c, $[A,b]$, and so on. In this situation, they speak on the y-stability, in particular, $[A,b,c]$-stability, b-stability, c-stability, and so on. Criteria for stability are formed via a certain set of properties of the mathematical object studied on its stability. Such criteria of stability mainly are

1) the boundedness of the admissible set;
2) the boundedness of the optimal set;
3) the stability of a certain set of inequalities-constraints (i. e., the conservation of the compatibility of the system under replacement of the inequalities $\le$ with the strict inequalities $<$).

The identification of the stability via the set of the notions cited above is sufficient. Naturally, a characterization of the stability is possible also via other systems of notions.

One can approach to the question of stability more widely. First, we can speak on a stability in the sense of conservation of a certain fixed property ω under small variations of the system of input data. In the linear case, this is $S_0 := \{a_{ji}, b_j, c_i\}$, Second, one can speak on a stability of the property ω, starting from variations of elements of an arbitrary subset S of the whole array of the input data S_0. Below the objects under investigation on stability will be

1) systems of linear inequalities;
2) linear programming problems.

In the capacity of ω, in the first of these two cases we consider the property of solvability or boundedness; in the second case, we take the boundedness of the optimal set. In doing so, we stress that we deal with the S-stability for arbitrary $S \subset S_0$.

In the situation of the stability defined in that way, we shall speak on (S,ω)-stability. Here a series of questions arise, each requiring a specific solution. First of all, it is the question on

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