Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 43, No. 12, pp.35–39, 1999 *UDC 519.8*

ON REGULARIZATION OF MULTICRITERIAL PROBLEM OF INTEGER LINEAR PROGRAMMING

V.A. Emelichev and O.A. Yanushkevich

As usual (see, e. g., [1]–[5]), in what follows a stability of multicriterial (vector) discrete optimization problem means that "small" perturbations of parameters of the initial problem do not cause appearing of new Pareto optimum. This property is a discrete analog of the upper semicontinuity in the Hausdorff sense (see [1], [6]–[8]) of the point-set mapping characterizing the dependence of the Pareto set on the initial data of a problem.

As known (see [1], [2], [4], [5], and [8]), scalar (one-criterial) linear problems of discrete optimization are always stable. The existence of non-stable (ill-posed in the Hadamard sense) vector problems of discrete optimization (see, e. g., [1]–[5]) yields the necessity for construction of a regularizing operator (if possible), which represents a special form of perturbation of the initial data of a discrete problem, in order to replace unstable problem with a series of perturbed stable problems, as it was made in the linear programming problem in [6], [9]. The first result of that sort was obtained in [10] (see also monograph [1]), where the regularization with respect to a vector criterion and to constraints of integer linear programming (ILP) problem on the search of the Pareto set on a bounded set of admissible solutions was constructed with the technique of convex cones. A regularization with respect to a vector criterion in [10] was based on replacement of a vector ILP problem by a perturbed problem with the Slater set contained in the Pareto set of the initial problem.

In the present article this result is generalized in the following sense: We construct a modified regularizing operator, which acts onto vector criterion and maps unstable initial vector ILP problem into a series of problems which are not only stable but also simultaneously equivalent, i. e., into a series of problems with the initial Pareto set. We propose also a way for ε-regularization of a vector ILP problem, which allows to replace the unstable problem by perturbed ε-stable problems.

Let a matrix $C = [c_{ij}]$ be in $\mathbf{R}^{m\times n}$, $m \geq 1$, $n \geq 1$. On a finite set of (admissible) solutions $X \subset \mathbf{Z}^n$ we define a vector linear criterion Cx. Its components (partial criteria) without loss of generality can be considered as being minimizing:

$$C_i x \to \min_{x\in X}, \quad i \in N_m.$$

Here $N_m = \{1, 2, \dots, m\}$, $x = (x_1, x_2, \dots, x_n)^T$, $C_i = (c_{i1}, c_{i2}, \dots, c_{in})$ and in what follows the subscript at a matrix (vector) indicates the corresponding line of the matrix (component of the vector).

Under the vector (m-criterial) ILP problem $Z^m(C)$ we mean the problem on determination of the Pareto set (the set of effective solutions):

$$P(C) = \{x \in X : \pi(x, C) = \emptyset\},$$

Supported by Belorussian Republic Foundation for Basic Research (grant F97-266).