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MAXIMUM PRINCIPLE IN NONSMOOTH PROBLEMS OF OPTIMAL CONTROL WITH DISCONTINUOUS TRAJECTORIES

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1. Introduction

The present article is devoted to the proof of necessary conditions for optimality in the form of the maximum principle (MP) for the problem of impulse control with a trajectory of the class BV of functions of bounded variation. The problem under consideration is characterized by a measurable dependence on time, the Lipschitzian dependence by phase coordinates, and the non-uniqueness of the reaction of the dynamical system on the impulse control, i. e., a vector measure. The last circumstance is related to the fact that we do not assume fulfillment of a well-posedness condition of the Frobenius type (see [1], [2]) which ensures the uniqueness of the trajectory corresponding to an impulse control and a given initial condition.

The problems of optimal impulse control can be treated as a relaxational expansion of classical problems of dynamical optimization in a system of the form

$$\dot{x} = f(t,x,V,u) + G(t,x,V)v, \quad \dot{V} = \|v\|, \tag{1}$$

$$u(t) \in U, \quad v(t) \in K, \tag{2}$$

where U is a compact, K is a convex closed cone, controls $u(\cdot)$, $v(\cdot)$ are measurable and bounded, $\|v\| = \sum\limits_{j=1}^{d(v)} |v_j|$, $d(z)$ stands for the dimension of the vector z, the second equation in (1) is introduced in order to take into account energy expenses for control. If the contrary is not stipulated, the terms "measurability" and "boundedness" applied to functions are related to the Lebesgue measure $\mathcal{L}$, while all relations containing measurable functions are assumed to be fulfilled $\mathcal{L}$-almost everywhere. Since the set K is unbounded, the optimization problems in system (1), (2) may have no solutions in the class of ordinary processes with controls from class L_∞ of measurable bounded operators and trajectories of the class AC of absolutely continuous functions. An intrinsic expansion of the problem is the result of taking the closure (in a certain weak topology) of a set of ordinary processes and is related to the notion of a generalized solution of system (1), (2).

Definition (see [3]). A pair of functions $(x(\cdot), V(\cdot))$ continuous from the right on $(t_0, t_1]$ and possessing bounded variation is called a generalized solution of system (1), (2) if a sequence of functions $\{x_n(\cdot),\ V_n(\cdot),\ u_n(\cdot),\ v_n(\cdot)\}$ exists satisfying on $[t_0, t_1]$ system (1), (2), such that

$$\sup_n \|v_n(\cdot)\|_{L_1} < \infty, \quad x_n \to x, \quad V_n \to V \ \text{ weakly}^* \text{ in } BV$$

(then $(x_n(t), V_n(t)) \to (x(t), V(t))$ at the continuity points x, V, and also for $t = t_0,\ t_1$).

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