Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 43, No. 12, pp.17–22, 1999 *UDC 519.8*

AXIAL THREE-INDEX ASSIGNMENT AND TRAVELLING SALESMAN PROBLEMS: FAST APPROXIMATE ALGORITHMS AND THEIR PROBABILISTIC ANALYSIS

E.Kh. Gimadi and A.I. Serdyukov

1. Introduction

Three-index axial assignment problem (3-AAP) (see [1]–[3]) is formulated as follows: Minimize the linear form

$$\sum_{i=1}^{n} c_{i,\pi(i),\sigma\pi(i)} \tag{1}$$

on the set of substitutions π, σ from symmetric group S_n of order n, where c_{ijk} are given real numbers, $1 \le i, j, k \le n$.

Along with this well-known assignment problem we consider the following new generalization of the classical travelling salesman problem.

Three-index axial travelling salesman problem (3-ATSP) represents a minimization of (1) with the additional condition

$$\pi, \sigma, \sigma\pi \in P_n, \tag{2}$$

where P_n is a set of all cyclic substitutions (i. e., consisting of one cycle) from S_n.

Theoretical results in [4]–[6] imply that both the problems are NP-complex; moreover 3-ATSP is MAX SNP-complex. This circumstance stimulated the authors to construct for these problems approximate polynomial algorithms on random inputs.

Fast approximate algorithms and probabilistic distributions on which these algorithms turn out optimal, represent a significant interest in the discrete optimization (see [7]–[14]).

We denote by f_A and f^* an approximate (obtained by algorithm A) value and an optimal value, respectively, of the objective function on a certain input. Recall that, in the case of concrete input data (problem's input), they usually speak on an individual problem. By a mass problem (or simply problem) we shall mean a definite set of individual problems.

In following [8], we shall say that an algorithm A *has estimates* (ε_A, δ_A) on a class of inputs of the problem under consideration if the inequality is valid

$$\Pr\{f_A > (1+\varepsilon_A)f^*\} \le \delta_A \ ,$$

where ε_A is the estimate of the relative error of solution obtained by the algorithm A, δ_A is the probability that algorithm A fails (i. e., the quantity δ_A can be treated as the part of cases when the algorithm does not guarantee an error lying within the limits ε_A).

It seems of interest to clarify the behavior of the estimates ε_A and δ_A under increase of problem's dimension.

Supported by Russian Foundation for Basic Research (codes of projects: 99-01-00601 and 97-01-00890).