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OPTIMIZATION OF DIRECT AND PROJECTION METHODS OF SOLVING OPERATOR EQUATIONS

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Introduction

Numerous problems of both theoretical and applied kind lead (see, e. g., [1]–[9] and references therein) to the necessity to solve singular integral equations (s. i. e.) of 1-st genus of the form

$$\frac{1}{\pi}\int_{-1}^{+1}\frac{x(\tau)\,d\tau}{(\tau-t)\sqrt{1-\tau^2}}+R(x;t)=y(t),\quad -1<t<1, \tag{0.1}$$

with the additional condition

$$\int_{-1}^{+1}\frac{x(\tau)\,d\tau}{\sqrt{1-\tau^2}}=0, \tag{0.1$'$}$$

the weakly s. i. e. of 1-st genus

$$\frac{1}{\pi}\int_{-1}^{+1}\frac{\ln|\tau-t|}{\sqrt{1-\tau^2}}x(\tau)\,d\tau+R(x;t)=y(t),\quad -1\le t\le 1, \tag{0.2}$$

and the singular integral differential equation of 1-st genus (so-called aerofoil theory equation) of the form

$$\frac{1}{\pi}\int_{-1}^{+1}\frac{\Gamma'(\tau)\,d\tau}{\tau-t}+V(\Gamma;t)=f(t),\quad -1<t<1, \tag{0.3}$$

under the boundary value conditions

$$\Gamma(-1)=\Gamma(+1)=0. \tag{0.3$'$}$$

Here R and V are given completely continuous (integral among them) or small by norm operators, $y(t)$ and $f(t)$ are given functions while $x(t)$ and $\Gamma(t)$ are the desired functions from some functional spaces.

As known (see [6], [10], [11]), the equations cited above usually cannot be solved exactly. This is the reason for development of numerous approximate methods (first of all, direct and projection ones) to solve them. In this connection a problem on optimization with respect to exactness of various classes of solving methods for such equations arises. A significant part of the present article is devoted to solution of this problem on the base of the results obtained earlier by the author on the optimization of methods of solving operator equations (see [2], [12], [13]) and on the direct methods (see, e. g., [2], [4], [5]) of solving equations (0.1)–(0.3). In this connection, in the beginning of the article we shall cite a series of results concerning the optimization of both direct and projection methods of solving linear operator equations in normed spaces.