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THEORY OF CURVES IN THE BIAFFINE FLAG SPACE OF HYPERBOLIC TYPE

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By a biaffine flag space of hyperbolic type, call it B_4^h, we shall mean a four-dimensional affine space whose fundamental group G consists of affine transformations preserving an absolute in the ideal hyperplane, and the absolute is that of the biaxial space of hyperbolic type (see [1]) together with a distinguished line of the absolute congruence. Note that B_4^h is a real model of the double biflag plane. The motion group of B_4^h depends on nine parameters. In the present article we develop the theory of curves in B_4^h.

1. For an appropriate choice of absolute, the group G of B_4^h is isomorphic to the matrix group

$$\begin{pmatrix} a_1 & b_1 & a_2 & b_2 & h_1 \\ \varepsilon b_1 & \varepsilon a_1 & \varepsilon b_2 & \varepsilon a_2 & h_2 \\ 0 & 0 & a_3 & b_3 & h_3 \\ 0 & 0 & \varepsilon b_3 & \varepsilon a_3 & h_4 \end{pmatrix},$$

where $\varepsilon = \pm 1$, $(a_3)^2 - (b_3)^2 = 1$.

Under the action of G the scalar product of vectors $\overline{a}(a_i)$, $\overline{b}(b_i)$ $(i = 1, 2, 3, 4)$,

$$\overline{a}\overline{b} = a_3 b_3 - a_4 b_4,$$

which is defined by a degenerate quadratic form of rank 2 and of index 1, is invariant, and the hyperbolic measurement of angles between m-planes $(m = 1, 2, 3)$ is invariant, too. Therefore B_4^h is a space with generalized projective metric (see [2]).

Two vectors are said to be orthogonal if their scalar product is zero. If the directions of orthogonal vectors determine points in the ideal hyperplane which lie on a line of absolute congruence, these vectors are called strongly orthogonal, and if a vector determines a point of the distinguished line, the vector is said to be isotropic.

It is evident that an isotropic vector is orthogonal to any nonisotropic vector. For two isotropic vectors $\overline{a}(a_i)$, $\overline{b}(b_i)$ we define a scalar product of second type, which is invariant under the action of G:

$$(\overline{a}\overline{b}) = a_1 b_1 - a_2 b_2.$$

If the second type scalar product of isotropic vectors is equal to zero, they are called isotropic orthogonal vectors.

Now let us consider a frame $R\{O, \overline{e}_i\}$ such that

1) O is an arbitrary point of the space;
2) $\overline{e}_1$, $\overline{e}_2$ are isotropic orthogonal, i.e., $\overline{e}_1^2 = \overline{e}_2^2 = 0$ and $(\overline{e}_1\,\overline{e}_2) = 0$;
3) $\overline{e}_3$, $\overline{e}_4$ are strongly orthogonal, and one of them is a unit vector while the other one is the imaginary unit, i.e., $\overline{e}_3\,\overline{e}_4 = 0$, $(\overline{e}_3\,\overline{e}_4) = 0$, $\overline{e}_3^2 = -\overline{e}_4^2 = \pm 1$.