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# ON CLASSES OF RELATIVE ALMOST HERMITIAN MANIFOLDS

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In the present article, by analogy with the classes of almost Hermitian manifolds (see [1], [2]), we introduce classes of relative almost Hermitian manifolds which are obtained from a general metric space of vector elements with relative metric. We find characteristics of these classes. The manifolds, tensor fields and objects are supposed to be differentiable of class $C^\infty$.

**1.** Let $M$ be a real $n$-dimensional manifold, $TM$ the tangent bundle of $M$ endowed with an infinitesimal (intrinsic by B.N. Shapukov, see [3]) connection $H$. Let $V$ be the vertical distribution, and $J$ an almost complex structure on $TM$ such that $J(X^h) = X^v$, $J(X^v) = -X^h$, where $X^h \in H$, $X^v \in V$. We denote by $\mathfrak{g}$ a metric of general metric space of vector elements $\mathfrak{g}_{n,y} = (M, \mathfrak{g})$, $y \in T_x$, $x \in M$, i.e., $\mathfrak{g}$ is a nondegenerate symmetric $M$-tensor field of type $(0,2)$ on $TM$ whose components are, by definition, relative of weight $w$ and homogeneous of a fixed degree $k$ with respect to fibre coordinates, e.g., the Moór metric (see [4]). Now let us assume that the Riemannian metric $\mathfrak{G}$ on $TM$ is induced by a positively definite metric $\mathfrak{g} : \mathfrak{G}(X^h, Y^h) = \mathfrak{G}(X^v, Y^v) = \mathfrak{g}(\tau_*(X), \tau_*(Y))$, $\mathfrak{G}(X^h, Y^v) = 0$, where $X = X^h + X^v$, $Y = Y^h + Y^v$, $\tau : TM \to M$. Then $\mathfrak{G}$ is a Hermitian metric whose components are relative of weight $W = \frac{1}{2}w$.

**Definition.** The almost Hermitian manifold $(TM, \mathfrak{G}, J)$ of the space $\mathfrak{g}_{n,y}$ will be called a relative almost Hermitian manifold (of weight $W = w/2$) or $A\mathfrak{H}$-manifold, for short.

We also introduce a relative torsion-free connection with covariant derivative $\nabla$ defined as a linear connection on $TM$ such that $\nabla\mathfrak{G} = 0$ for $2 + nw \neq 0$, $\nabla\mathfrak{G} = \mathfrak{G} \otimes \lambda$ for $2 + nw = 0$, and $\nabla_X Y - \nabla_Y X = [X, Y]$, where $\lambda$ is a recurrent covector field on $TM$, and $X$, $Y$ are elements of the module $\mathfrak{X}(TM)$ of vector fields on $TM$.

**2.** In [1], [2] some classes of almost Hermitian manifolds are listed and their characteristics are given. We shall consider analogous classes of $A\mathfrak{H}$-manifolds, just note that for relative almost Kählerian manifolds the fundamental 2-form $\mathfrak{F}(X, Y) = \mathfrak{G}(X, JY)$ of $A\mathfrak{H}$-manifold is not closed ($d\mathfrak{F} \not\equiv 0$) in general, unlike the case of almost Kählerian manifolds. The same thing occurs for locally conformally Kählerian structures (see [5]). Now let us list these classes of $A\mathfrak{H}$-manifolds and their characteristics.

(1) relative half-Hermitian $(H\mathfrak{H}) : \mathfrak{N}(X, Y, Z) + \mathfrak{N}(Y, X, Z) = 0$;
(2) relative semi-Hermitian $(S\mathfrak{H}) : \mathfrak{N}(X, Y, JZ) + \mathfrak{N}(Y, Z, JX) + \mathfrak{N}(Z, X, JY) = 0$;
(3) relative almost semi-Hermitian $(AS\mathfrak{K}) : (\delta\mathfrak{F})(X) = \lambda(JX)$;
(4) relative quasi-Kählerian $(Q\mathfrak{K}) : (\nabla_X J)Y + (\nabla_{JX} J)JY = 0$;
(5) relative almost Kählerian $(A\mathfrak{K}) : d\mathfrak{F} = (\lambda + \gamma W)\mathfrak{F}$ with a 1-form $\gamma$ on $TM$ such that $(\delta\mathfrak{F})(X) = \lambda(JX)$;
(6) relative nearly Kählerian $(N\mathfrak{K}) : (\nabla_X J)Y + (\nabla_Y J)X = 0$;
(7) relative Hermitian $(\mathfrak{H}) : N = 0$;
(8) relative semi-Kählerian $(S\mathfrak{K}) : (\delta\mathfrak{F})(X) = \lambda(JX)$, $N = 0$;
(9) relative Kählerian $(\mathfrak{K}) : \nabla J = 0$,