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FIRST INTEGRALS OF GEODESIC EQUATIONS OF H-SPACES OF TYPE [411]

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In the present article we find 6-dimensional h-spaces of type [411] and the first integrals of geodesic equations generated by projective transformations of these spaces. The general method of finding (pseudo)Riemannian spaces which admit nonhomothetic projective group G_r, was developed by A.V. Aminova (see [1]), and is described in detail in [2] for the case of h-spaces of type [51].

For an appropriate canonical frame $\{\underset{p}{\xi}{}^i\}$, the Eisenhart equations for an h-space of type [411] reduce to the equation system

$$
\begin{gathered}
X_r\lambda_4 = 0 \ \text{ for } \ r \neq 4, \quad X_r\lambda_\sigma = 0 \ \text{ for } \ r \neq \sigma, \quad X_4(2\lambda_4 - \phi) = X_\sigma(\lambda_\sigma - 2\phi) = 0, \\
\gamma_{314} = \frac{1}{2}e_4X_4\phi, \quad \gamma_{413} = \gamma_{431} = \gamma_{422} = -e_4X_4\phi, \quad \gamma_{1\sigma4} = \gamma_{2\sigma3} = \gamma_{3\sigma2} = \gamma_{4\sigma1} = \frac{e_4X_\sigma\phi}{\lambda_4 - \lambda_\sigma}, \\
\gamma_{2\sigma4} = \gamma_{3\sigma3} = \gamma_{4\sigma2} = -\frac{e_4X_\sigma\phi}{(\lambda_4 - \lambda_\sigma)^2}, \quad \gamma_{3\sigma4} = \gamma_{4\sigma3} = \frac{e_4X_\sigma\phi}{(\lambda_4 - \lambda_\sigma)^3}, \\
\gamma_{4\sigma4} = -\frac{e_4X_\sigma\phi}{(\lambda_4 - \lambda_\sigma)^4}, \quad \gamma_{4\sigma\sigma} = \frac{e_\sigma X_4\phi}{\lambda_4 - \lambda_\sigma}, \quad \gamma_{\sigma\tau\sigma} = \frac{e_\sigma X_\tau\phi}{\lambda_\sigma - \lambda_\tau};
\end{gathered}
\tag{1}
$$

here $X_p\phi = \underset{p}{\xi}{}^i\frac{\partial\phi}{\partial x^i}$, $\gamma_{pqr} = -\gamma_{qpr} = \underset{p}{\xi}{}_{i,j}\underset{q}{\xi}{}^i\underset{r}{\xi}{}^j$ are invariants, which play the role of the Ricci coefficients of rotation, ϕ is the determining function of projective motion, λ_4, λ_5, λ_6 are invariants such that $\lambda_\alpha \neq \lambda_\beta$ for $\alpha, \beta = 4, 5, 6$; $e_\alpha = \pm1$; $\sigma, \tau = 5, 6$.

The integrability conditions for system (1) can be written as follows

$$
\begin{gathered}
(X_1, X_2) = 0, \ \ (X_1, X_3) = 0, \ \ (X_2, X_3) = 0, \ \ (X_1, X_4) = e_3(\gamma_{314} - \gamma_{341})X_2, \\
(X_2, X_4) = -e_2\gamma_{242}X_3, \ \ (X_3, X_4) = e_3(\gamma_{134} - \gamma_{143})X_4, \ \ (X_1, X_\sigma) = -e_4\gamma_{4\sigma1}X_1, \\
(X_2, X_\sigma) = -e_3\gamma_{3\sigma2}X_2 - e_4\gamma_{4\sigma2}X_1, \ \ (X_3, X_\sigma) = -e_2\gamma_{2\sigma3}X_3 - e_3\gamma_{3\sigma3}X_2 - e_4\gamma_{4\sigma3}X_1, \\
(X_4, X_\sigma) = -e_1\gamma_{1\sigma4}X_4 - e_2\gamma_{2\sigma4}X_3 - e_3\gamma_{3\sigma4}X_2 - e_4\gamma_{4\sigma4}X_1 + e_\sigma\gamma_{\sigma4\sigma}X_\sigma, \\
(X_\sigma, X_\tau) = -e_\sigma\gamma_{\sigma\tau\sigma}X_\sigma + e_\tau\gamma_{\tau\sigma\tau}X_\tau.
\end{gathered}
\tag{2}
$$

By equating the coefficients at the derivatives $\frac{\partial}{\partial x^i}$ in the left-hand and the right-hand sides of (2) we obtain a system of partial differential equations. Then, by integrating this system, we get the components $\underset{p}{\xi}{}^i$ of the vectors with respect to the canonical frame. Using these results and the corresponding formulas (see [2]), we find the contravariant components of the metric tensor of the h-space under consideration, as well as the covariant components. We have