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DUAL AFFINE CONNECTIONS ON REGULAR HYPERBAND

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In the present article, with the use of connection theory in fibre bundles in the form developed by G.F. Laptev (see [1]–[3]), we study the intrinsic geometry of a normalized m-dimensional regular hyperband H_m immersed into an n-dimensional projective space P_n ($m < n-1$). Note that we use a minimally specialized reference system. From now on let us agree on the following index ranges

$$\overline{I}, \overline{K}, \overline{L} = \overline{0, n}; \quad \overline{i}, \overline{j}, \overline{k} = \overline{0, m}; \quad i, j, k, l, s, t = \overline{1, m}; \quad u, w = \overline{m+1, n-1}.$$

1. Let P_n be an n-dimensional projective space referred to a moving point frame $R \equiv \{A_{\overline{K}}\}$. The derivation formulas of R are $dA_{\overline{I}} = \omega_{\overline{I}}^{\overline{K}} A_{\overline{K}}$, where the Pfaff forms $\omega_{\overline{I}}^{\overline{K}}$ are subordinate to the structure equations of the projective space P_n (see [4], p. 143)

$$D\omega_{\overline{I}}^{\overline{K}} = \omega_{\overline{I}}^{\overline{L}} \wedge \omega_{\overline{L}}^{\overline{K}}, \quad \omega_{\overline{L}}^{\overline{L}} = 0.$$

As known (see [5], [6]), with respect to a first order frame the differential equations of regular hyperband $H_m \subset P_n$ ($m < n-1$) (see [7]) have the form

$$\begin{gathered} \omega_0^n = \omega_0^v = \omega_v^n = 0, \quad \omega_i^n = \Lambda_{ij}^n \omega_0^j, \quad \omega_i^u = \Lambda_{ij}^u \omega_0^j, \quad \omega_u^i = N_{uj}^i \omega_0^j, \\ \Lambda_{[ij]}^n = \Lambda_{[ij]}^u = 0, \quad \Lambda_{s[i}^n N_{|u|j]}^s = 0. \end{gathered} \tag{1}$$

Note that the system of functions $\{\Lambda_{ij}^n\}$ determines a first order tensor, and each of the function systems $\{\Lambda_{ij}^u, \Lambda_{ij}^n\}$, $\{N_{uj}^i\}$, $\{\Lambda_{ijk}^n, \Lambda_{ij}^n\}$ determines a second order differential object (see [2]), and the following equations hold true

$$\begin{gathered} \nabla\Lambda_{ij}^n + \Lambda_{ij}^n \omega_0^0 = \Lambda_{ijk}^n \omega_0^k, \quad \Lambda_{i[jk]}^n = 0, \\ \nabla\Lambda_{ij}^u + \Lambda_{ij}^u \omega_0^0 + \Lambda_{ij}^n \omega_n^u = \Lambda_{ijk}^u \omega_0^k, \quad \Lambda_{i[jk]}^u = 0, \\ \nabla N_{uj}^i + N_{uj}^i \omega_0^0 - \delta_j^i \omega_v^0 = N_{ujk}^i, \quad N_{u[jk]}^i = 0, \\ \nabla\Lambda_{ijk}^n + 2\Lambda_{ijk}^n \omega_0^0 + \Lambda_{(ij}^n \omega_{k)}^0 - \Lambda_{(ij}^n \Lambda_{k)s}^n \omega_n^s = \Lambda_{ijks}^n \omega_0^s, \\ \Lambda_{ij[ks]}^n = \Lambda_{it}^n \Lambda_{j[k}^w N_{|w|s]}^t + \Lambda_{tj}^n \Lambda_{i[k}^w N_{|w|s]}^t. \end{gathered} \tag{2}$$

Since the hyperband is regular, Λ_{ij}^n is nondegenerate, i.e., $\Lambda \overset{\text{def}}{=} |\Lambda_{ij}^n| \neq 0$; the first order relative invariant Λ satisfies the differential equation

$$d\ln\Lambda + m(\omega_0^0 + \omega_n^n) - 2\omega_k^k = \Lambda_k \omega_0^k, \quad \text{where} \quad \Lambda_k = \Lambda_n^{ji} \Lambda_{ijk}^n. \tag{3}$$

The components of the inverted tensor Λ_n^{ik} are defined via the relations

$$\Lambda_n^{ik} \Lambda_{kj}^n = \delta_j^i \tag{4}$$

and satisfy the differential equations

$$\nabla\Lambda_n^{ij} - \Lambda_n^{ij} \omega_0^0 = -\Lambda_n^{ik} \Lambda_n^{sj} \Lambda_{kst}^n \omega_0^t. \tag{5}$$
