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ON THE BEST NET, THE BEST SECTION, AND THE CHEBYSHEV CENTER OF BOUNDED SET IN INFINITE-DIMENSIONAL LOBACHEVSKIĬ SPACE

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In the present article we prove that some results due to A.L. Garkavi (see [1], [2]) and P.K. Belobrov (see [3], [4]) concerning the best net, the best section, and the Chebyshev center of bounded set in a special Banach space hold true for an infinite-dimensional Lobachevskiĭ space. Namely, for each bounded set in the infinite-dimensional Lobachevskiĭ space we prove: the existence of best N-net and best N-section; the uniqueness and the strong stability of Chebyshev center; the coincidence of the Chebyshev center with the Chebyshev center of closure of convex of a given set; the fact that the Chebyshev center lies in the closure of convex of a given set.

1. Preliminary. Definitions and theorems

Let V be a real Hilbert space, X the open unit ball centered at origin. In X we take the metric

$$\rho(x,y) = k\,\mathrm{Arch}\left[\frac{1-(x,y)}{((1-x^2)(1-y^2))^{1/2}}\right], \tag{1}$$

where (x,y) is the scalar product of vectors x and y in X, $k>0$. Thus we have obtained the well-known Beltrami–Klein model (see [5], p. 73) for the infinite-dimensional Lobachevskiĭ space.

Let us recall definitions and notation.

Let M be a bounded set in the space X. The radius of a covering of M by N-net $S_N = \{y_1,\dots,y_N\}$ $(y_n \in X)$ is the number defined as follows (see [1])

$$R(M,S_N) = \sup_{x\in M}\min_{1\le n\le N}\rho(x,y_n).$$

An N-net S_N^* such that $R(M,S_N^*) = \inf\limits_{S_N\subset X} R(M,S_N)$, where the infimum is taken over all N-nets in X, is called a best N-net for M (see [1]). A best 1-net y^* is called Chebyshev center of M, and $R(M) = R(M,y^*)$ is called the Chebyshev radius of M (ibid.).

A best N-dimensional section of a set M (unbounded, in general) in X is an N-dimensional plane H_N^* in X such that

$$\sup_{x\in M}\min_{y\in H_N^*}\rho(x,y) = \inf_{H_N\subset X}\sup_{x\in M}\min_{y\in H_N}\rho(x,y) < \infty, \tag{2}$$

where the infimum is taken over all possible N-planes in X (see [1]).

The Hausdorff distance between two bounded sets M and K in a metric space is the number $\delta(M,K) = \max\{\sup\limits_{x\in M}\rho(x,K); \sup\limits_{y\in K}\rho(y,M)\}$ (see [6], p. 441, [4]).

A sequence $\{M_n\}$ of bounded sets is said to converge in Hausdorff sense to a bounded set M if $\lim\limits_{n\to\infty}\delta(M_n,M) = 0$ (see [4]).

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