Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 11, pp.90–92, 1998 *UDC 512.533.5*

SEMIGROUPS WHOSE ALL NONCYCLIC SUBSEMIGROUPS ARE IDEALS

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Already for long time the study of algebras whose subalgebras possess a certain fixed property is among popular directions of algebraic investigations. In the theory of semigroups, the mentioned approach started since 50's and was developed by many authors. Let us mention several works which initiated our investigation. For example, in [1]–[3] authors independently describe semigroups in which any subsemigroup is an ideal (here and in what follows by an ideal we mean a two-sided ideal). On the other hand, in [4] the semigroups whose all proper subsemigroups are cyclic were characterized up to groups. Let us agree to call the semigroups with the first property *Id-semigroups*, while those with the second property — *C-semigroups*. If one has obtained a good description of algebras with a certain property for all subalgebras, it seems to be natural to extend this class, which frequently is reached by requiring that a fixed property must hold only for some subalgebras.

In the present article we study semigroups whose all noncyclic subsemigroups are ideals. We shall call such subsemigroups *IdNC-semigroups*. Clearly, any Id-semigroup and any C-semigroup are an $IdNC$-semigroup. In particular, so is any periodic C-group. The work [5] shows that the latter objects may possess very complicated structures. Using the description of Id-semigroups, we obtain a characterization of $IdNC$-semigroups. In this situation, the result of [4] about C-semigroups is not used; we derive this result as a corollary.

In the present article we use terminology and notation accepted in [6]. A subsemigroup H of the semigroup S is said to be *k-contracted by means of a subset M of the set $S \setminus H$* if $|M| \geq k$ and $\langle s_1, s_2, \ldots, s_k\rangle \supseteq H$ for any different elements $s_1, s_2, \ldots, s_k$ from M. If H is k-contracted by means of the set $S \setminus H$, then H will be simply called *k-contracted*. We should note that the term "contracted semigroup" was used in [7] in a different sense.

Let us define some types of semigroups, which we shall use in description of $IdNC$-semigroups.

Type 1. The semigroup S is 3-nilpotent, $S^2 = \{c, 0\}$, $a^2 = 0$ for a certain $a \in S \setminus S^2$ and S^2 is 2-contracted.

Type 2. Semigroup S is 3-nilpotent, $S^2 = \{a^2, c, 0\}$, the subsemigroup $\{c, 0\}$ is 2-contracted by means of the set $S \setminus S^2$ and $\langle x, y\rangle \supseteq S^2$ for any pairwise distinct indecomposable elements x and y, among which at least one is a divisor of the element a^2.

Type 3. Semigroup S is the amalgam of the semigroups A and B with a common subsemigroup $C = A^3 = B^2 = \{c, 0\}$, where A is the inflation of the cyclic nilpotent semigroup $\langle a\rangle$ of index 4 over the generator a; moreover, $c = a^3$, B is a nilpotent Id-semigroup of height 3 with 1-contracted subsemigroup C, and the operation of multiplication in S coincides on both A and B with the initial operations and for any elements $x \in A \setminus A^2$ and $y \in B \setminus B^2$ the relations $xy, yx \in C$ and $x^2y = yx^2 = 0$ take place.

Type 4. Semigroup S is the inflation of a cyclic nilpotent semigroup of index 4 over the generating element.

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