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SOLVING SINGULAR EQUATIONS WHICH WERE TRANSFORMED TO THE BEST ARGUMENT

Ye.B. Kuznetsov and V.I. Shalashilin

By singular systems of ordinary differential equations we shall mean systems consisting of ordinary differential equations and nondifferential relations. In the capacity of the latter we consider systems of nonlinear algebraic or transcendent equations. At the present time in the scientific literature to denote systems of that sort they frequently use the term "differential-algebraic" equations.

Apparently, numerical solving differential-algebraic equations was first investigated in [1]. A system of equations linear with respect to the derivatives $\dot{y}$:

$$A(y)\dot{y} + B(y) = f(t)$$

with a degenerated matrix $A(y)$, which describes processes in electric networks was integrated by means of the formulas of backward differentiation.

Later, for solving equations of this form, respective software was developed (see [2]–[5]). At the present time, along with numerous works concerning this problem some monographs were published (see [6]–[12]). However, in spite of undoubtful achievements in this domain, the results obtained earlier do not eliminate the difficulties of the numerical solution of differential-algebraic equations in comparison with the ordinary differential equations' solution (see [11]):

— the initial conditions must be consistent with nondifferential relations;

— a system of linear equations which is to be solved at each step of the integration process is badly conditioned for small steps; it was demonstrated (see [11]) that the conditioning of a system has the order $O(h^{\nu})$, where ν is system's index, h is the step of integration;

— the error of the method in the choice of integration step is sensitive to the inconsistency in the initial conditions and drastic changes of the solution;

— numerical solution depends in a great degree on the accuracy of approximation of the iteration matrix.

We introduce the following

Definition 1. A system of linear equations is said to be best conditioned if small changes of elements of the matrix of system or its right side lead to least change of the solution.

In the present article we suggest a method by which the solution of the Cauchy problem for a system of differential-algebraic equations is considered from the standpoint of the method of continuation of the solution along the parameter, which allows to pose the question on the choice of the best parameter (see [13]–[16]) which supplies the best conditioning to the system of linear equations of the continuation. This leads to weakening of a part of difficulties mentioned above. Thus, a system of linear equations of the continuation, obtained at each step of integration process, will be the best conditioned. Moreover, by virtue of the choice of argument of the problem, the error will be less sensitive to drastic changes of the solution.

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