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A MIXED FINITE-ELEMENT METHOD IN THE PROBLEM ON EIGENVALUES OF NONLINEAR STABILITY OF SLANTING SHELLS

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In the present article we continue the investigation of the convergence of a mixed finite-element method in problems of stability of slanting shells, which was started in [1]. In contrast to [1], we shall consider here a nonlinear stability problem in the form of a quadratic problem on eigenvalues. The latter is reduced to a spectral analysis of a quadratic selfadjoint bundle of completely continuous operators. The estimates for rate of convergence of eigenvalues and eigenvectors of a discrete problem, constructed via the Herman–Johnson scheme of the mixed finite-element method, are obtained.

1. Initial assumptions and statement of the problem

Let Ω be a convex polygonal domain from R^2 with the boundary $\partial\Omega$. We define the spaces

$$M = [L_2(\Omega)]^3, \quad H = [H_0^1(\Omega)]^3, \quad V = [H_0^1(\Omega)]^2 \times H_0^2(\Omega)$$

with their norms $|u|_0^2 = |u_1|_{0,\Omega}^2 + |u_2|_{0,\Omega}^2 + |u_3|_{0,\Omega}^2$, $\|u\|_1^2 = \|u_1\|_{1,\Omega}^2 + \|u_2\|_{1,\Omega}^2 + \|u_3\|_{1,\Omega}^2$, and $\|u\|_V^2 = \|u_1\|_{1,\Omega}^2 + \|u_2\|_{1,\Omega}^2 + \|u_3\|_{2,\Omega}^2$, respectively.

Suppose that displacements of the middle surface of the initial equilibrium state of a shell $\overline{u}$ in a certain neighborhood of the supposed critical state have been computed or estimated, and deformations in this neighborhood can be linearized. Then one can introduce a load parameter λ so that the linearized deformations of the critical state will be expressible via $\lambda\overline{u}$. Under these assumptions, the variational problem of nonlinear stability of slanting shells with the homogeneous Dirichlet conditions for bifurcation components of displacements on the boundary $\partial\Omega$ consists in the following (see [2], [3]): Find a pair $(\lambda, u) \in C \times V$ such that $u \neq 0$ and

$$a(\nabla_2 u_3, \nabla_2 v_3) + c(u, v) = \lambda d_1(u, v) + \lambda^2 d_2(u, v) \quad \forall v \in V. \qquad (EP)$$

Here

$$a(\nabla_2 u_3, \nabla_2 v_3) = \int_\Omega D_M[\partial_{11}u_3\partial_{11}v_3 + \partial_{11}u_3\partial_{22}v_3 + \partial_{22}u_3\partial_{11}v_3 + \partial_{22}u_3\partial_{22}v_3 - \\ - (1-\nu)(\partial_{11}u_3\partial_{22}v_3 + \partial_{22}u_3\partial_{11}v_3 - \partial_{12}u_3\partial_{12}v_3)]dx,$$

$$c(u, v) = \int_\Omega D_N\Big[\varepsilon_1(u)\varepsilon_1(v) + \varepsilon_2(u)\varepsilon_2(v) + \\ + \nu\varepsilon_1(u)\varepsilon_2(v) + \nu\varepsilon_2(u)\varepsilon_1(v) + \frac{1-\nu}{2}\varepsilon_{12}(u)\varepsilon_{12}(v)\Big]dx,$$

$$d_1(u, v) = \int_\Omega \sum_{i,j=1}^{2} [N_{ij}^l(\overline{u})\partial_i u_3\partial_j v_3 + N_{ij}^l(u)\partial_i\overline{u}_3\partial_j v_3 + N_{ij}^l(v)\partial_i\overline{u}_3\partial_j u_3]dx,$$