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THE UNIQUENESS OF POSITIVE SOLUTION OF A CERTAIN TWO-POINT BOUNDARY VALUE PROBLEM WITH THE DIGITAL SCHEME FOR ITS EVALUATION

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Let us consider on the segment $[0,1]$ the following two-point boundary value problem:

$$y'' + ax^m y^n = 0, \quad 0 < x < 1, \tag{1}$$

$$y(0) = y(1) = 0. \tag{2}$$

Here m, n, and a are constants such that $a > 0$, $m > 0$, also $n > 1$. By a positive solution of problem (1)–(2) we shall mean its solution in the class $C^2[0,1]$, which satisfies the inequality $y(x) > 0$ with arbitrary $x \in (0,1)$.

In this article we shall prove the existence and uniqueness of a positive solution of problem (1)–(2) and propose a non-iterative way for its numerical evaluation. The existence of a positive solution of problem (1)–(2) can be proved also by means of S.I. Pohozhaev's stratification method (see, e. g., [1], [2]).

1. A priori bounds

Theorem 1.1. *A positive solution $y(x)$ of problem (1)–(2) satisfies the estimations*

$$M_1 a^{-\frac{1}{n-1}} \le \max_{[0,1]} y(x) \le M_2 a^{-\frac{1}{n-1}}, \tag{1.1}$$

where M_1 and M_2 are positive constants depending only on m and n.

Proof. Let $M = \max\limits_{[0,1]} y(x) = y(d)$, $d \in (0,1)$. Then $y'(d) = 0$, $y''(d) \le 0$. We have

$$M = \int_0^d y'(t)dt = -\int_0^d ty''(t)dt = a\int_0^d t^{m+1}y^n(t)dt. \tag{1.2}$$

On the other hand,

$$M = -\int_d^1 y'(t)dt = -\int_d^1 (1-t)y''(t)dt = a\int_d^1 (1-t)t^m y^n(t)dt. \tag{1.3}$$

Since $y''(x) < 0$ for $x \in (0,1)$, we have that $y(x)$ is a convex upwards function. Consequently, $y(t) \ge \frac{y(d)}{d}t = \frac{M}{d}t$ for $0 \le t \le d$ and $y(t) \ge \frac{1-t}{1-d}y(d) = \frac{M(1-t)}{1-d}$ for $d \le t \le 1$. By means of these bounds from (1.2) we obtain

$$M \ge \frac{aM^n}{m+n+2}d^{m+2}. \tag{1.4}$$