Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 9, pp.63–66, 1998 *UDC 514.76*

ON A CLASS OF THREE-WEBS OF TYPE $W(4,4,2)$ WITH CONSTANT COMPONENTS OF THE FUNDAMENTAL TENSOR

Nguen Zoan Tuan

1. By definition, a three-web of type $W(P,P,Q)$, $P>Q$, is a triple of foliations on a $(P+Q)$-dimensional differentiable manifold M whose leaf families S_a, $a=1,2,3$, consist of surfaces of the dimensions P, P, and Q, respectively, and have the property that through each point $x\in M$ only one surface V_a from each family passes, where x is the unique point of intersection of the surfaces V_1 and V_3, and the same is true for the surfaces V_2 and V_3.

In [1] the general case of three-webs of type $W(p,q,r)$ was considered, while in [2] — the three-webs of type $W(n-1,n-1,1)$. The latter one is a special case of the three-webs of type $W(P,P,Q)$, $P>Q$.

Let us consider the bundle M' over M consisting of the frames $\{x,e_i,\overline{e}_i,e_\alpha\}$, $i=\overline{1,Q}$, $\alpha=\overline{Q+1,\dots,Q+P}$, where at each point x the vectors e_i are tangent to the surface of the first family, $\overline{e}_i$ are tangent to the surface of the second family, $e_i+\overline{e}_i$ are tangent to the surface of the third family, and e_α are simultaneously tangent to the surfaces of both the first and the second families. Then M' is a G-structure on M, where G is a subgroup of the general linear group which consists of the matrices (cf. [3])

$$(U^J_K)=\begin{pmatrix} U^\beta_\alpha & 0 & 0\\ U^\beta_i & U^k_i & 0\\ -U^\beta_i & 0 & U^k_i\end{pmatrix}$$

$J,K=1,\dots,n=P+Q$; $i,k=1,\dots,Q$; $\alpha,\beta=Q+1,\dots,Q+P$, $\det(U^\beta_\alpha)\neq 0$, $\det(U^k_i)\neq 0$.

To each frame $\{x,e_i,\overline{e}_i,e_\alpha\}$ the coframe of dual linear forms ω^i, $\overline{\omega}^i$, ω^α corresponds. These forms are said to be basic, and their linear combinations are called the principal forms.

The families S_1, S_2, S_3 can be given via the following completely integrable Pfaff equation system

$$\begin{aligned} \overline{\omega}^i&=0 && \text{(the equations of a surface in } S_1);\\ \omega^i&=0 && \text{(the equations of a surface in } S_2);\\ \omega^i-\overline{\omega}^i&=0,\quad \omega^\alpha=0 && \text{(the equations of a surface in } S_3). \end{aligned}$$

The conditions of complete integrability of these equations constitute the fundamental structure equations of the three-web, which can be reduced to the form (cf. [3])

$$\begin{aligned} d\omega^i&=\omega^i_k\wedge\omega^k+A^i_{lk}\omega^l\wedge\omega^k+A^i_{\alpha k}\omega^\alpha\wedge\omega^k,\\ d\overline{\omega}^i&=\omega^i_k\wedge\overline{\omega}^k+A^i_{lk}\overline{\omega}^l\wedge\overline{\omega}^k+A^i_{\alpha k}\omega^\alpha\wedge\overline{\omega}^k,\\ d\omega^\alpha&=\omega^\alpha_k\wedge(\omega^k-\overline{\omega}^k)+\omega^\alpha_\beta\wedge\omega^\beta, \end{aligned}\tag{1}$$

where $A^i_{lk}+A^i_{kl}=0$.