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ON MINIMAL SUBMANIFOLDS OF CONSTANT CURVATURE IN EUCLIDEAN SPACE

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Connected minimal surfaces of constant Gaussian curvature were classified in [1]. Isometric immersions of regions in n-dimensional Lobachevskiĭ space L^n into a $(2n-1)$-dimensional Euclidean space E^{2n-1} were considered in [2]–[4]. In [2] the following result was proved: In the normal bundle of L^n in E^{2n-1} a basis exists with zero torsion coefficients. In [3] coordinates in L^n were introduced with the use of asymptotic directions. In [4] it was proved that on a region $D \subset L^n$ which is isometrically immersed into E^{2n-1} one can take local coordinates whose coordinate lines are the curvature lines. In [5] isometric immersions of L^n into E^{n+m}, $m \geq n-1$, were considered being such that the normal connection of immersion is flat, or, equivalently, at each point of the immersed region $D \subset L^n$ n principal directions exist. Using these principal directions, one can take orthogonal coordinates $u_1, \dots, u_n$ in D such that the line element has the form $ds^2 = \sum\limits_{i=1}^{n} \sin^2 \sigma_i du_i^2$. Let us take an orthonormal basis of normal fields ξ_p, $p = 1, \dots, m$, whose torsion coefficients are zero. With respect to this basis the second fundamental forms of D are diagonal: $\mathrm{II}^{(p)} = \sum\limits_{i=1}^{n} L_{ii}^{(p)} du_i^2$, $p = 1, \dots, m$. In [5] it was proved that with each isometric C^3-immersion of a region of L^n into E^{n+m}, $m \geq n-1$, whose normal connection is flat, one can associate the following orthogonal $(m+1) \times (m+1)$-matrix

$$\begin{bmatrix} \sin\sigma_1 & \dots & \sin\sigma_n & H_{n+1} & \dots & H_{m+1} \\ \dfrac{L_{11}^{(1)}}{\sin\sigma_1} & \dots & \dfrac{L_{nn}^{(1)}}{\sin\sigma_n} & \Phi_{n+1\,1} & \dots & \Phi_{m+1\,1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dfrac{L_{11}^{(m)}}{\sin\sigma_1} & \dots & \dfrac{L_{nn}^{(m)}}{\sin\sigma_n} & \Phi_{n+1\,m} & \dots & \Phi_{m+1\,m} \end{bmatrix}.$$

Note that $\sum\limits_{i=1}^{n} \sin^2 \sigma_i \leq 1$. Our main result is the following

Theorem. *A region D in the Lobachevskiĭ space L^n cannot be isometrically C^3-immersed into an Euclidean space E^{n+m}, $m \geq n-1$, as a minimal submanifold with flat normal connection.*

Remark. For $m = n-1$, the requirement for the normal connection to be flat is superfluous and we thus get another proof of theorem 2 from [3] in the case of Euclidean ambient space.

First, let us prove the following

Lemma. *For the mean curvature vector H of the submanifold $L^n \subset E^{n+m}$ the following equality takes place* $H^2 = \frac{1}{n^2} \sum\limits_{i=1}^{n} \frac{1}{\sin^2 \sigma_i} - 1$.