Russian Mathematics (Iz. VUZ) *Izvestiya VUZ. Matematika*
Vol. 42, No. 9, pp.52–60, 1998 *UDC 517.958*

AN EXPLICIT METHOD OF SECOND ORDER ACCURACY FOR SOLVING STIFF SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS

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1. Introduction

In the present article we consider an explicit numerical method for solving systems of ordinary differential equations. Explicit methods have both positive and negative features. Among positive ones we can cite the following: they are simple to realize, can be easily parallelized, and require relatively small memory. In [1], [2], steps varied with respect to time are used, connected via certain relations with the roots of Chebyshev's polynomial. This results in first order schemes where the average step by time is higher than in schemes with a constant step. In [2]–[6], methods of the second order accuracy by time were also considered. The problem of construction of methods of that sort is reduced to construction of polynomials possessing properties of the Chebyshev alternance on a definite given segment and approximating the exponent with the second order accuracy in a neighborhood of zero. This problem can be reduced to a construction of polynomials with least deviation from zero (see [2], [5]) with a weight. In [5], [6], numerical methods for construction of such polynomials were considered. In [3], [7] it was shown that many of these polynomials can be expressed via Zolotaryov's polynomials. Applying these results, one can formulate the problem of construction of explicit methods of second order with an enlarged stability domain and determine parameters of these methods, which are expressed via the roots of Zolotaryov's polynomials. The parameters of these polynomials are found. At the end of Section 4 we supply tables of roots; via these roots the parameters of an explicit method of second order with maximal real stability domain are expressed.

2. Some notions and definitions

Let $y(t) = (y_1(t), y_2(t), \ldots, y_n(t))$ $(n \geq 1)$ be a vector column with coordinates independent of the time t. Consider the following Cauchy problem: For $t_0 \leq t \leq T$ find $y(t)$ as a solution of the problem:

$$\frac{dy}{dt} = f(y,t), \quad y(t_0) = y_0, \tag{1}$$

where $y_0 = (y_{01}, y_{02}, \ldots, y_{0n})$ is a given vector.

Let

$$J(t) = J(y,t) = ||f_i/y_j|| \tag{2}$$

be the Jacobi matrix of $f(y,t)$; in it $y(t)$ is the solution of (1).

Supported by Russian Foundation for Basic Research, grant 96-01-01481. Graphics are produced with the use of graphical library kindly granted by Professors G. Wanner and E. Hairer.