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ANALYTIC SOLUTIONS OF PROBLEMS ON OSCILLATIONS OF ELASTIC PLATES AND SHELLS WITH REINFORCEMENTS OF APPLICATIONS TYPE

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In [1], a general statement of problems of dynamic deformation of thin elastic shells under loads distributed along squares of fixed dimensions was given. Further development of an approach suggested in [1] was achieved in [2], where the problem on common oscillations of a thin elastic shell of arbitrary shape and a solid body adjoined to the plate was considered. In [3], a general statement of dynamic contact problem was given for an elastic shell of arbitrary shape. Under given kinematics of the motion of points which belong to a fixed contact region, a generalized solution of the problem was written. In the class of generalized functions, the exact solutions of a series of dynamic and static problems of contact interaction between thin elastic shells, moment-free cylindric shells and absolutely solid bodies were constructed.

In the present article we suggest a description of a new method for solving problems on oscillations of thin elastic shells with rigid applications. Based on applying generalized functions, this method allows to take into account the mass and geometry of an application, and also a manner in which it is fixed on a shell. The method suggested here is applied to the investigation of oscillations of a round cylindric shell with an absolutely rigid ring-shaped application of a finite width. Both the direct and inverse statements of the problem are considered. Exact analytic solutions are found.

1. *Description of the method.* Let us represent the mathematical model of a dynamic behavior of a thin elastic shell with an application by means of the following operator equation

$$A\ddot{\overline{U}}(\alpha_1,\alpha_2,t)+C\overline{U}(\alpha_1,\alpha_2,t)=\overline{F}(\alpha_1,\alpha_2,t),\quad (\alpha_1,\alpha_2)\in S, \tag{1.1}$$

where A and C are the inertial and elastic operators, respectively, defined over a class of functions satisfying the boundary conditions on the shell's edge, $\overline{U}$ is the vector function which determines, for instance, displacements and the rotation angles of a cross-cut section, $\overline{F}$ is the vector function determining exterior action on the shell, α_1 and α_2 are Gaussian coordinates of the points of middle surface, t is the time, S is a simply connected domain occupied by the middle surface of the shell.

Assume that the application is stiffly fixed on one of the face surfaces of the shell. Suppose also as being known the law of displacement of an arbitrary point of the application along the spatial coordinates α_1, α_2

$$\overline{U}(\alpha_1,\alpha_2,t)=\overline{U}_0(\alpha_1,\alpha_2,t),\quad (\alpha_1,\alpha_2)\in S_\sigma, \tag{1.2}$$

where S_σ is the contact region, which posses a piecewise-smooth boundary L.