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ON THE REDUCIBILITY OF NONLINEAR DIFFERENTIAL EQUATIONS

E.V. Voskresenskiĭ

In [1], the concept of the Lyapunov transformation was generalized for nonlinear differential equations and sufficient conditions for the reducibility of equations from certain classes were found. In the present article we shall give a reducibility criterion which generalizes the known Erugin theorem (see [2], p. 154) and formulate new reducibility conditions based on this criterion.

Let Ξ be a set of differential equations of the following form

$$\frac{dx}{dt} = f(t, x), \tag{1}$$

where $f \in X$, $f(t, 0) \equiv 0$, and $X = C^{(p,q)}([T, +\infty) \times R^n, R^n)$ is the space of all vector-functions $(t, x) \to 1(t, x)$ of dimension n, which are defined on the set $[T, +\infty) \times R^n$, p times continuously differentiable with respect to variable t, $p \geq 0$, and q times continuously differentiable with respect to components of the vector x, $q \geq 1$. We denote by $x(t : t_0, x_0)$ the solution of equation (1), which satisfies the initial condition (t_0, x_0). We suppose that all the solutions $x(t : t_0, x_0)$ are determined for all $t \geq T$ for any equation from the set Ξ.

First, let us extend the concept of a Lyapunov transformation introduced in [1].

Definition. A group of transformations $G = \{\varphi : \varphi : \Xi \to \Xi\}$ is called the Lyapunov group of transformations (LG, Ξ) if the characteristical exponents and the stability of the trivial solution are invariant. If φ belongs to a certain Lyapunov group (LG_1, Ξ^1), $\Xi^1 \subseteq \Xi$, then φ is called the Lyapunov transformation and the corresponding equations are said to be mutually reducible.

We denote by Ξ_1 the set of all equations (1) satisfying the additional property $\|f(t, x)\| \leq \psi(t)\|x\|$, where $\psi \in C([T, +\infty), [0, +\infty))$ and the function ψ depends on the function f. Assume that $\Xi_1 \subset \Xi$.

Theorem 1. *The group G_2 of all transformations $\varphi : \Xi \to \Xi$ such that*

1) *any its transformation is a function $x = \varphi(t, y)$ satisfying the conditions $\varphi \in C^{(p_0,q_0)}([T, +\infty) \times R^n, R^n)$, $p_0, q_0 \geq 1$, $\|\varphi(t, y)\| \leq k_0\|y\|$, $k_0 > 0$ for all $t \geq T$;*
2) *any inverse function $y = \varphi^{-1}(t, x)$ satisfies the conditions $\varphi^{-1} \in C^{(p_0,q_0)}([T, +\infty) \times R^n, R^n)$ and $\|\varphi^{-1}(t, x)\| \leq k_1\|x\|$, $k_1 > 0$ for all $t \geq T$, $x \in R^n$,*

is the Lyapunov group (LG_2, Ξ).

Proof. The inequalities $\|\varphi(t, y)\| \leq k_0\|y\|$, $\|\varphi^{-1}(t, x)\| \leq k_1\|x\|$ imply that characteristical exponents and the stability of the trivial solution are invariants for the group G_2. Consequently, this group is the Lyapunov group (LG_2, Ξ). □